THE IMPACT OF USING SYNCHRONOUS COLLABORATIVE VIRTUAL TANGRAM IN CHILDREN'S GEOMETRIC

Chiu-Pin LIN,
National Hsinchu University of Education
Taiwan
chiupin.lin@gmail.com

Yin-juan SHAO
Learning Sciences Laboratory, National Institute of Education
Singapore
yj_shao@hotmail.com

Lung-Hsiang WONG,
Learning Sciences Laboratory, National Institute of Education
Singapore
lhwong2000@yahoo.com

Yin-Jen LI
Digital Education Institute, Institute for Information Industry
Taiwan
yinjen@iii.org.tw

Jitti NIRAMITRANON
Faculty of Engineering, Kasetsart University
Thailand
jitti@jitti.com

ABSTRACT
This study aimed to develop a collaborative and manipulative virtual Tangram puzzle to facilitate children to learn geometry in the computer-supported collaborative learning environment with Tablet PCs. In promoting peer interactions and stimulating students' higher-order thinking and creativity toward geometric problem-solving, we designed a collaborative Chinese Tangram activity with problem-solving learning strategies. Participants are 25 6th graders of a suburb elementary school of Tai-Chung City. The results suggest that children's competency in rotation and space of shapes had been improved and the scores gap between lower and higher achievers had been narrowed. Such a collaborative Chinese Tangram activity may facilitate peer negotiation, enhance children's belief toward problem solving, and benefit each child to share resources, and a positive interdependent learning context can naturally be developed.

INTRODUCTION
In geometry teaching, Tangram puzzle could be used as an aid in presenting specific mathematical concepts, inspiring children's observation, imagination, shape analysis, creativity and logical thinking (e.g., Lee, et al., 2008; Olkun, Altun & Smith, 2005; Russell & Bologna, 1982; Sedig, Klawe & Westrom, 2001; Yang & Chen, 2010). It allows children to develop geometric concepts by categorizing, comparing and working out the puzzle, and thereupon to solve problems in geometric contexts. The proliferation of Information and communications technology (ICT) has added new facets to the limited physical learning resources by creating virtual learning aids. In the meantime, collaborative learning could also be supported by Internet, mobile technology and handheld devices. In traditional tangram games, individuals often played the game by him/herself. Even people played the game as a group, disorders often occurred when everybody lent a hand at those pieces. Pieces scrabbling might result in quarrels and fights and slow down the speed of completion. The virtual tangram was found to solve these problems. On the other hand , playing the virtual tangram game offered by Group Scribbles 2.0, a novel networked collaborative learning environment (see below for more details), could help students develop collective cognition and collaborative skills when play the game together and share with other groups. We conducted an experimental study with 25 Grade 5 (11-year-old) students in an elementary school. The objective of this research was to investigate whether the students can develop mathematics concepts through playing the Tangram puzzle collaboratively, and solving problems together through discussion. This article is structured as follows: The next section is a review of the literature. The third section describes the research design. The results for the various analyses are presented in the latter sections. Finally, conclusions are presented and suggestions are made for further research.

LITERATURE REVIEW

Geometry and spatial ability

Learning Geometry is a process of studying the conversion of graphics in the space, which can enhance children's spatial ability (Zhou, 1999; Do & Lee, 2009). Van Hiele (1986) proposed a five-level model describing how people learn Geometry. These levels are a product of experience and instruction, moving from visualization, analysis, abstraction, deduction to rigor. Children in the 6th grade of elementary school generally reach level 1 and start to move to level 2. At this stage, students should have gone beyond identifying basic geometric figures and analyze the properties of graphic, but also learn to recognize the relationships between types of shapes. Playing the Tangram puzzle is one of the significant methods to enhance geometric spatial thinking. The Tangram puzzle has been used in prior studies focused on computer supported learning (Scarlatos et al., 2002; Sedighian & Klawe, 1996). Actions of dissection, rearrangement and recomposing promote the imagination and logical thinking through observation and analysis (Clements & Battista, 1992).

Mathematical problem solving skills

Given emphasis on developing skills of mathematical problem solving in NCTM (National Council of Teachers of Mathematics) standard, peer work and discussion in the process of exploration are referred and strongly recommended (NCTM, 1990). Many researchers applied collaborative learning strategies to enhance students' motivation and interest as well as their skills of mathematical problem solving (Ho, 2007; Liu, 1990). Working as a group and asking each group member to share their conversation has therefore been proven effective to approach the specific goal (Huang & Wu, 2006). Interactions among group members, collaboration and idea exchanges could give a push to deep thinking, exploration, reasoning and problem solving (e.g., Coleman, 2008).

One-to-one digital learning

One-to-one (1:1) technology enhanced learning refers to the setting of one or more computing device per student in a wireless networking environment for learning. The devices used for such a learning mode usually incorporate the following affordances: (1) portability; (2) supporting social interactivity; (3) personalization; (4) context sensitivity; (5) connectivity; and (6) bridging the cyberspace and the physical world (Chan et al., 2006). In 1:1 digital learning, instructors could make their choices of the most suitable devices for their teaching and learning purposes. Students' learning and interactive modes show greater variety in 1:1 digital learning classrooms (Roschelle, 2003; Zurita & Nussbaum, 2004a; Looi et al., 2009). The use of Tablet computer directs collaborative learning to a new horizon with the affordances of mobility and handwriting recognition. Mobility advances the interaction among group members on the move by keeping materials visible and usable to the whole group. One Tablet PC per person provides equal opportunities to each student to exchange ideas, sharing resource and gains immediately (Zurita & Nussbaum, 2004b).

Virtual Tangram

Virtual manipulations were regarded as a simulated learning tool on computers, allowing students to move, turn and rotate virtual objects on the screen through manipulating the keyboard and the mouse (Moyer, Bolyard & Spikell 2002; Sedig, 2008). In this study, we adopted virtual Tangram puzzle to support learning. Students were asked to discover the rules and the relationships of graphics in geometry by playing virtual Tangram puzzle, recomposing and representing the visuals. This would help students concentrate more on thinking and finish the composition more precisely.

Most current virtual learning aids could not satisfy the purpose of learner-centered learning for discussion and collaboration because of the lack of affordances for collaborative learning activities. With the rapid advancement of ICT, handheld devices and Internet communications offer real-time interaction for learning (Zurita & Nussbaum, 2004). Hence, the collaborative virtual learning aids could offer more new directions in supporting geometric learning which we were keen to explore in this study. The virtual Tangram puzzle inherits the advantages of traditional tangram but with more advanced features of remote playing and less risk of losing pieces. The result of virtual tangram puzzle could also be easily shared not only within a group but also inter-groups, by only projecting the screen for public view. Students can share their composition results of tangram for each other.

METHODS

Twenty five Grade 6 (11-year-old) students (15 males, 10 females) from an elementary school in Tai-Chung City, Taiwan took part in this study. For analyzing different ability students' learning achievement and collaborative strategies during the progress. They were first divided into eight groups of high-, medium- and low-ability according to the pre-test scores. There were 3 students in each of those seven groups and 4 students in the last group. The eight consist of two high-ability groups (Group 1 and Group 2), two medium-ability groups (Group 3 and Group 4), two low-ability groups (Group 5 and Group 6), and two mixed groups with high-, medium- and low-ability students (Group 7 and Group 8).

Group Scribbles (GS2.0), a computer-supported collaborative learning system developed by SRI International, was adopted to conduct small group collaborative for their puzzle- shape solving (Chaudhury et al., 2006; Looi, Lin & Liu, 2008). The interface of Group Scribbles in the figure 1 contains three main parts: public board, group board and private board. The teacher can add, move and adjust the boards to his/her needs. The virtual Tangram puzzle was added to the toolkits. Each piece of the Tangram puzzle had numbers on it and can be rotated and moved freely.

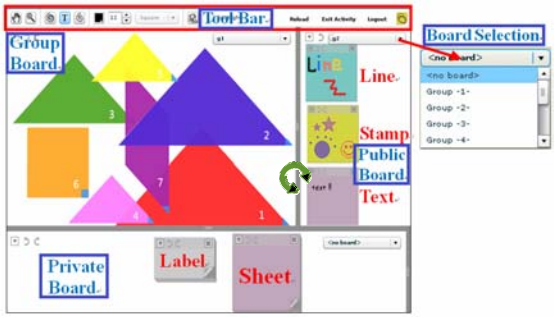

Figure 1. Interface of GS 2.0

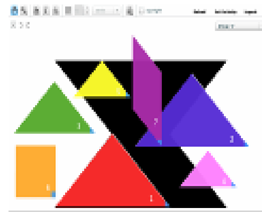
Figure 2. Task 1

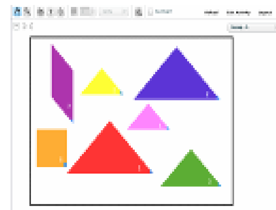
Figure 3. Task 2

Experimental Process

To develop geometric spatial sense and creative thinking, the researchers designed two tasks according to the two puzzle modes. The experiment lasted for four weeks and the process was as below:

1. Pre-test (10 minutes): The paper-and-pencil test was administered for students.
2. Group Scribbles warm-up practice (30 minutes): The teacher introduced to the students the electronic notes, the shared space and the concept mapping tool to familiarize them with the GS environment. The students then practiced to make a square with the Tangram puzzle set by rotating and moving each piece of the Tangram puzzle freely.
3. Perform the task 1 (40 minutes): Sudents in each group were required to make a funnel within 8 minutes (Figure 2).The results of each group were shared in the class and the teacher made some interpretation. During the classes, the teacher monitored the activities of all the groups and provided assistance where necessary.
4. Perform the task 2 (40 minutes): All groups practiced piecing up a sailing boat. Again 8 groups had a competition to complete a 'sailing boat' collaboratively within 8 minutes. Results were shared and peer assessment took place.
5. Post-test and questionnaire (40 minutes): The 10-minute post-test was administered promptly at the end of the intervention, which was followed by the administration of the questionnaire, which lasted another 30 minutes.
6. Student interviews: Semi-structured one-to-one interviews with ten students were then conducted in the final week.

The qualitative data consisted of students' interviews that sought their perceptions in learning, and videos that recorded the whole process of intervention analyze their in-group interactions.

RESULTS

Data analysis

The researchers analyzed the results of the pre- and post-tests, collecting and comparing the lowest and highest score, mean and the Standard Deviation.

Table 1: Average score and SD in Pre and Post tests

	Lowest score	Highest score	Average score	SD	t	Sig.
Pre-test	26	100	61.60	23.16	-3.564	.001**
Post test	40	100	73.28	18.99		

Table 1 shows that there has been an increase of the lowest score, from 26 to 40. The overall mean was 11.68 points higher than the pre-test. The difference of the mean between pre and post-test is significant ($p=0.001 <.01$), which means that the target students had significantly improved their mean.

The researchers performed further analysis on the statistics by executing paired sample t-tests on their pre- and post-test results respectively.

Table 2: Pre- and Post- test results of four groups

Group	Average		SD	t	Sig.
High-ability group (n=6)	Pre-test	83.00	8.17	-0.542	0.305
	Post-test	84.67	6.28		
Medium-ability group	Pre-test	70.33	14.39	-0.810	0.227

(n=6)	Post-test	73.00	20.15		
Low-ability group (n=6)	Pre-test	39.33	6.77	-2.775	0.020*
	Post-test	58.00	19.80		
Mixed group (n=7)	Pre-test	54.86	28.21	-2.940	0.013*
	Post-test	76.86	19.14		

Table 2 depicts that all groups scored higher in the post-test. Among these groups, the low-ability groups (p=0.02 <.01) and the mixed groups (p=0.013 <.01) had significantly improved their scores, catching up with the high and medium-ability groups.

To examine their learning gains in the target domain, the researchers had also conducted paired sample t-tests to the questions in pre- and post- tests according to various dimensions of geometric competencies. The competency dimensions are: basic shape rotation (Figure 4), spatial reasoning and sensing in Tangram puzzle (Figure 5), and piecing together the combinational shapes (Figure 6).

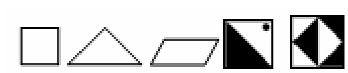

Figure 4. Basic shapes

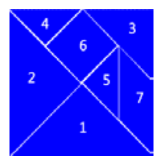

Figure 5. Spatial reasoning and sensing

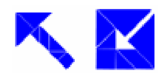

Figure 6. Combinational shapes

Questions 1 to 5 were about the rotation of basic shapes. For example, “What graphic will it be when we rotate one of the figure 4 right 45 º?”

Table 3: The Competency of basic shape rotation in pre- and post- tests

Rotation of basic shapes	Lowest score	Highest score	Average	SD	t	Sig.
Pre-test	8	40	25.60	10.07	-2.347	.014*
Post-test	8	40	30.08	9.60		

Table 3 illustrates that the students received the score of 30.08 in the post-test, which is 4.48 higher than that in the pre-test. It also indicates that the students had enhanced their competency in basic shape rotation after the experiment (p=0.014 <0.5).

Questions 6 to 10 were about spatial reasoning and sensing with the Tangram puzzle set. For example, “in Figure 5, which of the piece has the same size as the piece No. 6?”

Table 4: The competency of spatial reasoning and sensing in pre- and post- tests

Spatial reasoning with tangram	Lowest score	Highest score	Average	SD	t	Sig.
Pre-test	0	40	24.32	10.95	-2.828	.005*
Post-test	8	40	30.72	8.54		

As it demonstrated in Table 4, the mean of students in the post-test measured 6.4 points higher than that in the pre-test and the p value is 0.005, reached the 0.05 level. From the statistic figures of Questions 1 to 5 and Questions 6 to 10, the changes in Questions 6 to 10 are more significant. This indicates participants had made good progress in the competency in spatial reasoning in this experiment and better understood the feature of Tangram.

Question 11 and 12 were about piecing the combinational shapes with the Tangram puzzle set. For example, “What does figure 6 come to be when it is turned left 90 º?”

Table 5: The competency of piecing combinational shapes in pre- and post tests

Combinational shapes composition	Lowest score	Highest score	Average	SD	t	Sig.
Pre-test	0	20	11.60	8.00	-0.768	.225
Post-test	0	20	12.80	7.37		

Table 5 shows that participants did make improvement and scored higher, though the difference was not significant (p=0.225). This indicates that children in the experiment didn't promote much in their competency in piecing combinational shapes.

In short, the collaborative puzzle shaping with the Tangram puzzle can enhance the shapes rotation and spatial ability. Furthermore, the students' competency in spatial reasoning and sensing had also been improved. Nonetheless, it did not show much help in developing the competency in piecing combinational shapes as no significant difference between the pre and post-tests. More exercises may be needed to develop the patterns of spatial capability with further in-depth study.

Patterns found in the process of collaborative tangram playing

During the process of this experiment, five patterns were found out from analysis of field notes and video recordings.

(1) Communication in the group: It was identified that the students had carried out a mixture of GS-based and face-to-face communications. Two people sitting next to each other often turned aside to watch one screen together when they had discussion. The third participant sitting opposite to those two, as illustrated in Figure 7, moved him/herself to the other side to view the same screen. This indicates all group members were engaged in the discussion. They carried out face-to-face discussion as well as online communications.

(2) Ways of collaboration: When a member in the group had difficulties in handling the Tangram puzzle on his/her computer, other members would move themselves to give help and make demonstration, as illustrated in Figure 8. Such behaviors helped to establish the sense of positive interdependence and sharing of failure and success, thus improving the collaboration. Gesticulating on the screen could be regarded as the replacement of actual pieces grabbling in traditional tangram. Within a group this physical behavior could be a great aid for verbal discussion.

Figure 7. Group members' communication

Figure 8. Group members' mutual help

(3) Sharing the achievement: One delegate of each group was asked to interpret their patterns of puzzle shaping and present the achievements of the group (Figure 10). Figure 9 illustrates the results of each group's work. Students may find out other patterns of composition constructed by other groups on the public board of GS. By showing all groups' results on the public board through projecting all students can discuss the work within the whole class, improving their individual knowledge through the collective knowledge.

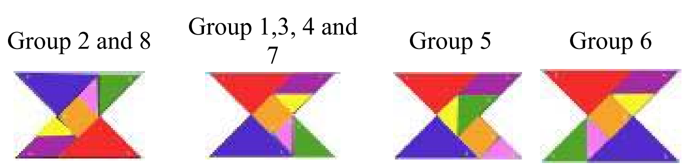

Figure 9. Comparison of different composition patterns

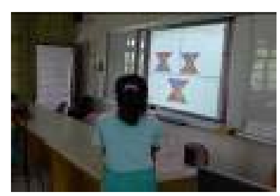

Figure 10. Group delegate present

(4) Discussion: The teacher explained the concept of poly forms, the same size in different shapes. The reflection of their problem-solving process would increase students' understanding to the Tangram puzzle. In addition, the teacher interpreted the area concept of square, parallelogram and quadrilateral. As shown in Figure 11, the teacher could demonstrate the concept of area by both manipulating the virtual Tangram puzzle as well as drawing shapes on the board. The teacher found it easy to summarize and explain the geometrical concept based on students' own collective cognition and knowledge.

(5) Peer group assessment: After Task 2, all students in the class did a peer assessment to other groups with 'sheet' and 'stamp' in GS in the scale of 1 to 5. The result of their peer assessment is displayed in Figure 12. Each group could easily obtain peer feedback in this way. That may encourage students to make rigorous observation which will lead to critical thinking.

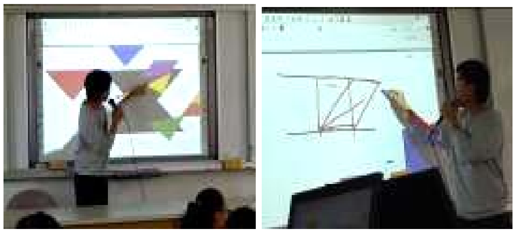

Figure 11. The teacher's interpretation

Figure 12. The result of peer assessment

Findings from the post-questionnaire

A post-questionnaire of the experiment was administered and analyzed according to Likert's five-point scale standard (Strongly agree, agree, Neutral, disagree and strongly disagree). We investigated the five facets of the experiment to analyze the students' perceptions and experiences in the Tangram puzzle activities and found positive results.

Table 6: Findings from questionnaires

Item	Mean	SD
Usability		
*1. It was difficult to get used to the virtual Tangram puzzle	2.84	0.8
2. It was easy to operate the system	3.48	1.05
3.I was not stuck in the process of using the system	3.76	0.97
Strategies in problem solving		
4. I would like to piece the puzzle after the discussion with my partners	3.92	1.00
5. I would like to sketch the shape in advance	4.20	0.76
6. I would scrabble the puzzle after deep thinking	4.04	0.79
7. I would ask for help from my partners if I encountered difficulties	3.88	0.78
8. I would keep trying new patterns	4.16	0.69
Engagement		
9. I felt well engaged in those activities	4.48	0.82
10. I would try my best to complete the puzzle with the discussion with my partners	4.36	0.76
11. I would give as much feedback as possible to my partners	4.40	0.71
12. I would try my best to get the goals of our group	4.40	0.71
Learning activities		
13. My partners and I were easy to reach a consensus	3.48	1.09
*14. My partners were not willing to discuss with me	2.48	1.12
15. My partners and I gave each other feedback	4	0.81
16. I had learned more knowledge about the geometric graphics	4.16	0.85
17. I could complete the puzzle shapes together with my partners	4.40	0.76
18. I could successfully complete the shape that I want	3.92	0.91
*19. It had no help in my learning	2.08	1.12
Motivation and interest		
20. It was more interesting to piece the puzzle with my partners	4.12	0.83
21. I found it quite interesting to play virtual Tangram puzzle	4.40	0.82
*22. I found it easier to play the Tangram puzzle by my self	2.84	1.14
23. I liked to discuss problems with my classmates	3.96	0.68
24. I would like to have another play, managing other shapes	4.52	0.87
25. I hope we can have more activities like this in our Math class	4.24	1.17

(1) Usability: The mean of Question 1 and Question 2 are 2.84 and 3.48 respectively. Only 12% of the students were not familiar with the touch pen and it took time for them to get used to. During the post-interviews that followed, the students complained that their peers in other group move their Tangram puzzle in the process. We should consider it as a point to improve the system.

(2) Strategy in problem solving: All students had applied different strategies such as problematical thinking, trial and error, and peer help searching to achieve their goal except differed in ways of discussion.

(3) Engagement: Students were very engaged in puzzle piecing and could help each other. More than 80% of the students held positive view to collaborative puzzle shaping and struggled for the goal.

(4) Learning activities: Some groups lacked collaborative skills which had resulted in conflicts. 20% of the students felt being isolated or not being able to communicate with their group mates whenever a particular group member started dominating the group. However, 76% of the participants believed that playing virtual Tangram would stimulate more idea exchange and interaction with their group mates. In general, over 70% of students could complete the puzzle shapes and enhance their understanding of geometric graphics. Over 90% of the students could reach the goal through collaboration despite that 12% of them were less actively engaged in the activities and held negative views.

(5) Motivation and interest: More than 70% of the participants stated that they liked to work with their group mates to solve problems. Over 80% of them anticipated alternative challenge of puzzle shaping and expected more interesting collaborative activities like that. This indicates their strong willingness of learning through playing the tangram and consequently they were fully motivated to learn.

Findings from the interviews

Semi-structured interviews were conducted to investigate the student experience and perceptions in collaborative problem solving with puzzle piecing as well as the process of learning. The findings from the interviews are analyzed from three perspectives. (NB: 'Gx' represents the group and 'Sxx' represents the student)

Impact of collaboration in problem solving: The group strategy to ask children to challenge jigsaw puzzle as a group demonstrated that in the cooperative process of discussion and sharing students also created and exchanged ideas, had their group work skills improved through interaction, thereby had promoted the motivation and interest of learning.

"Everybody could think together, which may contribute to the understanding of Tangram puzzle"（G2-S9）

"We carried out group discussions if we couldn't work it out"（G6-S20）

"We knew more ways of puzzle piecing"（G1-S23）

"We could distribute the work and reach the goal with collaboration"（G2-S9）

"It is more interesting to play the Tangram puzzle with others than to play all by myself"（G6-S20）

Learning experience: Playing Tangram puzzle collaboratively allows students to obtain empirical experiences. Students held positive views to the cooperative Tangram puzzle. They claimed that they could play puzzle with group members easily and which is more interesting than individuals. But some of the students had difficulty to use the touch pen, which get in the way to organize and compose the shape freely.

"It deepened the impression with the practical operation" (G6-S6-23)

"We could stop anytime where necessary by using computers and keep going after full understanding"（G5-S7）

"I was engaged to play the Tangram puzzle with my classmates and discuss how to solve the problems"（G1-S16）

"Very interesting, I want more Tangram puzzle"（G2-S9）

"The touch pen is not easy to handle. I hope I can use the mouse instead" (G5-S21)

Learning Gains: By rotating and moving each piece of the Tangram puzzle, it helps students to understand the geometry and size composition. Not only can review students' previous mathematics knowledge but also stimulate their imagination.

"I gained a better understanding in geometric graphics after playing the Tangram puzzle"（G6-S20）

"I understood the size of the dismantling and the composition and decomposition of graphics"（G7-S4）

"It helped to understand the approaches of calculation"（G7-S4）

"It helped to develop my imagination by scribbling "（G2-S9）

CONCLUSIONS

In this study, a 1:1 digital learning environment for computer supported collaborative learning was facilitated to the students for learning geometric concepts. Each student was given equal opportunity to operate the system for developing the geometric spatial sensing and reasoning skills. The researchers argued that the collaborative learning activity of virtual Tangram could reduce the gap between high-ability and low-ability students. The empirical operation could help in understanding the composition of area and stimulate the imagination and creativity. What's more, the collaborative virtual learning tool enabled resource sharing and formed the interdependent learning environment. Partners in the group can help each other by thinking together, discussing and giving each other feedback. They also learned how to negotiate with their partners and had their confidence strengthened to problem solving with better motivation and interest.

The findings of this research could be possible reference for relevant research and practices while some further suggestions need to be addressed as below:

1. Expand the sample of target group and set up control group for further research.
2. Prolong the implementation and enable sufficient time for exercises.
3. Add more projectors to each group or the space of public board to promote face-to-face interaction.
4. Methods of mixed heterogeneous grouping can promote students learning with different ability to learn from each other and solve problems collaboratively.
5. The teacher should train students and develop their group work skills.
6. The teacher should encourage all students to participate in the activity and overcome difficulties with time scheduling and multi-resources of groups.

REFERENCES

Chan, T.W., Roschelle, J., Hsi, S., Kinshuk, Sharples, M., Brown, T., Patton, C., Cherniavsky, J., Pea, R., Norris, C., Soloway, E., Balacheff, N., Scardamalia, M., Dillenbourg, P., Looi, C., Milrad, M.., & Hoppe, U. (2006) One-to-one technology-enhanced learning: An opportunity for global research collaboration. *Research and Practice in Technology Enhanced Learning,* 1(1), 3-29.

Chaudhury, S. R., Roschelle, J., Schank, P., Brecht, J., & Tatar, D. (2006). Coordinating student learning in the collaborative classroom with interactive technologies. *Poster presented at the 3rd International Society for the Scholarship of Teaching & Learning Conference 2006*, Washington D.C. USA.

Clements, D. H., & Battista, M. T. (1992). Geometry and spatial reasoning. In: Douglas, A. G. (Ed.), *Handbook of Research on Mathematics Teaching and Learning* (pp.420-464). New York: Macmillan.

Coleman, J. L. (2008). *The development of understanding of the concept of variable in grade seven beginning algebra students: the role of student interaction*. Master Dissertation, Kingston, Canada: Queen's University.

Do, T. V., & Lee, J.-W. (2009). A multiple-level 3D-LEGO Game in augmented reality for improving spatial ability. *Proceedings of the International Conference on Human-Computer Interaction* (pp. 296-303). San Diego, CA.

Ho, G.-L. (2007). A cooperative learning programme to enhance mathematical problem solving performance among secondary three students, *The Mathematics Educator,* 10(1), 59-80.

Huang, Z., & Wu, J. (2006). *The Development and Practice of Collaborative Learning.* Taibei: Wunan publisher

Lee, C.-W., Huang, J.-K., Chou, W.-S., Sun, H.-H., Yeh, T.-Y., Huang, M.-J., & Chen, H.-C. (2008). Development of a geometry learning game with tangible user interfaces. *Proceedings of the World Conference on Educational Multimedia, Hypermedia and Telecommunications 2008* (pp. 1548-1553). Vienna, Austria.

Liu, X. (1990). *The research and practice of teaching patterns of mathematic problem solving by collaborative reflection.* PhD thesis, Taipei, Taiwan: National Taiwan Normal University.

Looi, C. K., Lin, C. P., & Liu, K. P. (2008). Group Scribbles to Support Knowledge Building in Jigsaw Method. *IEEE Transactions on Learning Technologies*, 1(3), 157-164.

Moyer, P. S., Bolyard, J. J., & Spikell, M. A. (2002). What are virtual manipulatives? *Teaching Children Mathematics,* 8(6), 372-377.

NCTM (1990). *Curriculum and Evaluation Standard for School Mathematics (3rd Ed.)*. Reston, VA: NCTM.

Olkun, S., Altun, A., & Smith, G. (2005). Computers and 2D geometric learning of Turkish fourth and fifth graders. *British Journal of Educational Technology*. 36(2), 317-326.

Roschelle, J. (2003). Unlocking the learning value of wireless mobile devices. *Computer Assisted Learning,* 19(3), 260-272.

Russell, D. S., & Bologna, E. M. (1982). Teaching Geometry with Tangrams. *Arithmetic Teacher*, 30(2), 34--38.

Scarlatos, L., Landy, S., Breban, J., Horowitz, R., & Sandberg, C. (2002). On the effectiveness of tangible interfaces in collaborative learning environments. *Proceedings of SIGGRAPH Conference 2002* (pp. 125-126), San Antonio, TX.

Sedighian K., & Klawe, M. (1996). Super Tangrams: A childcentered approach to designing a computer supported mathematics learning environment. *International Conference on Learning Sciences 1996*) pp. 490-495), Evanston, IL.

Van Hiele, P. M. (1986). *Structure and insight: A theory of mathematics education.* Orlando, FL: Academic Press.

Yang, J.-C., & Chen, S.-Y. (2010). Effects of gender differences and spatial abilities within a digital pentominoes game. *Computers in Education,* 55(3), 1220-1233.

Zhou, S. (1999). *New Mathematics Teaching Methods for Children.* Taipei: Psychology publisher.

Zurita, G., & Nussbaum, M. (2004a). Computer supported collaborative learning using wirelessly interconnected handheld computers. *Computers & Education,* 42(3), 289-314.

Zurita, G., & Nussbaum, M. (2004b). A constructivist mobile learning environment supported by a wireless handheld network. *Computer Assisted Learning,* 20(4), 235-243.