

IPAD INFUSE CREATIVITY IN SOLID GEOMETRY TEACHING

Shih-Hwa Liu

Department of Information Management
National Taiwan University of Science and Technology
Taipei, Taiwan, R.O.C.
nelsontaipei@gmail.com

Gwo-Guang Lee

Department of Information Management
National Taiwan University of Science and Technology
Taipei, Taiwan, R.O.C.
lgg@mail.ntust.edu.tw

ABSTRACT

We unveiled our plans to revolutionize the students' spatial conception development through the challenge and support of a cooperative learning of practice, the development of the profession as a whole and through sharing innovation and expertise. This encompasses cognitive consultancy, curriculum integration, solutions architecture, management of cooperative learning, embedding the solution and delivering through efficient technology involvement. This unique difference means we can advise on the implementation of those technologies within a live environment. In Taiwan, we are experimenting with a high level of technology integration to suit the student's learning requirements in the solid geometry curriculum. Married to this idea is the use of the iPad. The implementation of a new teaching model using the iPad has been very successful. The devices have been well received by students and by teachers and are increasingly well-used in the solid geometry curriculum as their attributes and limitations are learned. Both pedagogical changes and new ways of learning engender by access to information and learning tools, progress in the implementation of the scheme has been outstanding. By having teachers become more aware of their students' cognitive skills, attitudes, and misconceptions, teaching practices and student achievement can be enhanced.

INTRODUCTION

Solid geometry is an important part of the mathematics curriculum, and being the foundation for study in such fields as graphics, geology, science, engineering, architecture, computer and astronomy (Banchoff, 1990; Senechal, 1990). However, students are not demonstrating strong conceptual knowledge in reasoning about geometric ideas (Carroll, 1998). Carroll (1998) found that students are capable of developing continued growth and power of reasoning. The visualization of geometry improves students' perception of spatial relationships. Unfortunately, many students develop misconceptions, and others fail to derive from geometric figures. Learners who have engaged with geometry are likely to have an example space based on their previous encounters with geometry (Mason, 2003).

In applying the view of Piaget (1959) to how students learn within a dynamic geometry software environment, the students would construct their mathematical knowledge through interaction with the software, building their ideas through interaction and reflection on the results of their actions, a process facilitated by the feedback provided by the computer. This places implications on the learning environment itself. The layout of most school computer rooms positions the PC around the perimeter of the room or in blocks facing each other. It is not unusual for students learning in such a space to be completely silent. The teacher is paramount in creating an atmosphere that allows the students to interact in an environment that promoted collaborative inquiry about concepts using appropriate tools.

Recently, we have relied upon the mobile learning as a way to infuse the technology of the computer and software to the learning objectives found in the various content areas of our curriculum. Staying up to date on new technology, teacher can incorporate geometry software on class to present students the geometry graphics dynamically. When students are poorly prepared in critical order thinking skills and independent inquiry skills, the development can be supported by effective use of mobile technologies. A considerable body of literature confirms there has been a significant and very positive impact on learning which be reflected in the achievement and attainment (Hwang, Chu, Lin, & Tsai, 2011; Hwang, Wu, & Ke, 2011). Like the iPod and iPhone, the iPad is a platform intended to support online content including books, magazines, games, music, and video, plus Web access. Because of the bigger display, it is potentially a better platform for mobile learning than the iPhone or the iPod Touch. With the iPad, teachers can offer a flexible, complete, and cost efficient alternative to the traditional classrooms. Great iPad when teachers need to quickly explain something that requires a sketch or drawing and it's easy to share to cloud storage.

The iPad and the Cabri3D software can combine to provide an advantageous alternative to traditional solid geometry teaching. Certainly merging into an iPad environment from a traditional classroom can be challenging. We should explore the instructional implications of a move towards this new option at the Taiwan high school. Besides, we will address a gap in the literature by examining the impact on learning and teaching in an innovative school that already has a strong commitment to ICT. In particular, it will focus on changes in teaching and learning styles, impact on standards and on student's attitudes to learning with the iPad. Melhuish and Falloon (2010) lead us to a consideration of how mobile technologies are redefining what constitutes a learning space, one that is no longer fixed in time but based on connecting people with each other and information through virtual collaborative spaces and communities.

We proposed a new teaching model, is based on the theories of Van Hiele and Vygotsky. Theoretical foundations such as constructivism, social interaction and discussions, the use of smartboard, iPads, Cabri3D, cooperative learning groups, and higher order thinking are just a few of the theories that support our teaching philosophy and this research. For all the sophistication of our technology, our view of learning is still talking about courses and investigate students' learning of solid geometry in a phase-based instructional environment using iPad based on the van Hiele theory. This study was undertaken to investigate the following research questions,

1. Is the new teaching model useful in promoting students' thinking processes on geometry tasks?
2. Can the levels be characterized operationally by student behavior?
3. Can an interview procedure be developed to reveal different levels of reasoning on specific geometry tasks?

We use technology to move from an event-based learning model that we know to be ineffective, to a more distributed and contextualized environment that spans the continuum from formal learning to performance support.

THEORETICAL BACKGROUND

van Hiele theory

The van Hiele theory of geometric thinking comprises three main components; levels of geometric thinking, characteristics of the levels and phases of learning (Crowley, 1987). The van Hiele defined five phases of reasoning in geometry and the role of instruction in raising levels of thinking. The student passes before jumping to the next level (van Hiele-Geldof, 1957). They believed the developmental model of thought processes were discrete through which student progress as they learn geometry.

1. Level 1 (Visualization). The student reasons about geometric figures, such as simple shapes, by identifying, naming and comparing them according to their appearance. Perception is visual only and primarily by means of visual considerations of the concept as a whole without explicit regard to the properties of its components.
2. Level 2 (Analysis). The student reasons about geometric concepts empirically, such as folding, measuring, analyzing figures in terms of their components and relationships among components. Necessary properties and their attributes are used to describe and established figures.
3. Level 3 (Abstraction). The student logically orders previously discovered properties of concepts, form abstract definitions. By giving informal arguments the student operates with these concepts both within a figure and between related figures.
4. Level 4 (Deduction). The student can manipulate the relationships within the context of a mathematical system rather than a collection of shapes. Reasoning at this level includes complete with undefined terms, axioms, an underlying logical system, definitions, and theorems.
5. Level 5 (Rigor). The student can analyze and compare systems based on different axioms. The student can study various geometries in the highly abstract and does not necessarily involve pictorial models. At this level the axioms themselves become the object of intense rigorous scrutiny.

The van Hiele levels have generally been accepted by some researchers as a reasonable explanation as to how students learn geometry (Wilson, 1990; Flores, 1993; Battista, 1994; Perdikaris, 1996; Sharp & Hoiberg, 2001). In addition to furnishing insights into the thinking that is specific to each level of geometric thought, the van Hiele identified some generalities that characterize the model. These properties are particularly significant for educators because they provide guidance for making instructional decisions.

1. Sequential. As with most developmental theories, a person must proceed through the levels in order. To function successfully at a particular level, a student requires adequate and effective learning strategies in order to learn how to think and reason at higher levels (Crowley, 1987; Clements & Battista, 1993).
2. Advancement. Progress from level to level depends more on the influence of a teaching-learning program than on age. No method of instruction allows a student to skip a level; some methods enhance progress, whereas others retard or even prevent movement between levels (Crowley, 1987; Corley, 1990).
3. Intrinsic and extrinsic. The inherent geometric concepts that implicitly understand at one level become explicitly understood at the next level (Corley, 1990).
4. Linguistics. Each level has its own language, set of symbols and network of relations (Crowley, 1987; Clements & Battista, 1993).
5. Mismatch. If the student is at one level and instruction is at a higher level, the desired instruction and progress may not occur. The teacher, instructional materials, content, vocabulary is at a higher level than the learner, the student will not be able to understand the thought processes being used (Crowley, 1987).

Despite its importance, students still performed poorly on the compulsory solid geometry questions in Mathematics. To help students progress from one level to the next, the van Hiele propose five sequential phases of learning: inquiry, directed orientation, explication, free orientation, and integration (van Hiele-Geldof, 1984).

1. Information. The teacher and students engage in conversation and activity about the objects of study for this level. Questions are raised, and observations and evaluations are made how level-specific vocabulary is interpreted. And gives them some awareness of why they are studying the topic, so as to set the stage for further study.
2. Directed orientation. The students explore the topic of study by doing short tasks designed that the teacher carefully sequenced. These activities should gradually elicit the students the structures characteristic of this level. These steps help students acquaint themselves with the objects from which geometric ideas are abstracted.
3. Explication. Students learn to express and exchange their emerging views about the structures that have been observed during class discussions. The teacher leads students' discussion of the objects of study in their own words, so that students become explicitly aware of the objects of study. Then, the teacher introduces what figures and properties emerged in the activities above vocabulary.
4. Free orientation. The student encounters more complex tasks - tasks with many steps, tasks that can be completed in different ways. The teacher encourages students to solve and elaborate on these problems and their solution strategies.
5. Integration. The students review and summarize what they have learned about the objects of study with the goal of creating an overview of the topic. The teacher guides students through this process using standard vocabulary, but does not present any new ideas. At the end of the fifth phase, the students have attained a new level of thinking about the topic of study.

Sabean and Bavaria (2005) examined the research, which suggested that instruction must be balanced between the practice of skills and methods previously learned and new concept discovery. This discovery of new concepts, they suggest, facilitates a deeper understanding of mathematical connections. Researchers have explored the impact of technology on geometry that reported by GeoCAL (Chang, Sung & Lin, 2007) , DALEST developing an Active Learning Environment for Stereometry (Clements, Battista, Sarama & Swaminathan, 1997) , Dynamic Geometry Systems (DGS) and the Geometric Supposers (Schwartz & Yerushalmy, 1992). Some scholars analyze characteristics and properties of geometric shapes and develop mathematical arguments about geometric relationships based on the ideas of NCTM (National Council of Teachers of Mathematics) (Christou, Jones, Pitta-Pantazi, Pittalis, Mousoulides, Matos, Sendova, Zachariades & Boytchev, 2007).

Vygotsky theory

Vygotsky (1978) took up the idea of the Zone of Proximal Development (ZPD) as the difference between the level of difficulty of a subject that the student can understand with the help of a teacher or a fellow student. The concept of ZPD emerged as an argument against the use of standardized tests to gauge the human intelligence within our society or culture. The learner proceeds to the next developmental level of participation in activities slightly beyond their competence.

Vygotsky's theory of individual intellectual development emphasizes the importance of individual cognitive gains occur first through interaction with the social environment and then is internalized in the individual (Vygotsky, 1978; Rogoff, 1990). It is contrasted to Piaget's supposed tendency to view learning as a primary private affair. Based on Vygotsky's work, the learning communities can support learning through assisted performance, managed discourse, and reciprocal teaching.

Rather than the push to memorize facts, cognitive psychology advanced beyond behaviorism by positing that students actually could try to find facts and how to distinguish solid from the environment. Practical experiences involve problem solving situations that cognition has gone beyond knowledge to recognize not only operate “in the wild”, but use information in the environment as part of the thinking process (Hutchins, 1996). Some students this age still confuse solid shape.

By assessing prior knowledge, the teacher is in accordance with both the constructivist view of teaching and learning mathematics will be able to see where the students are cognitively and push to have them work in their zone of proximal development through scaffolding (Vygotsky, 1978; Vygotsky, 1986). Teachers create an attractive scenario and assign students homework to on a daily basis. There is all manner of grunt work that students need to do in terms of time spend interacting with others. They can’t really get rid of much of that grunt work, but it can streamline it. Playful activities in which students can physically practice directional instructions help them develop a kinesthetic understanding of solids. That offers a value to the students whenever they achieve new levels. Once these processes are internalized, they become part of the student's independent developmental achievement (Vygotsky, 1978).

METHODOLOGY

Computer Software

Current some math software is designed for plane geometry use and is poorly suited to solid geometry. The Cabri3D has become the dominant tool for giving students a tangible, visual way to explore and understand core concepts of geometry (See Fig 1). The Cabri3D's friendly user interface allows teachers and students to get quickly up to speed so teacher can spend time on teaching mathematics, not software. The teacher can easily generate dynamic instructional materials with accurately measured figures by exporting Cabri3D files to word and PowerPoint programs, and the internet. Teachers can provide engaging learning experiences and explore variables, relationships, and the mathematics of change with their students.

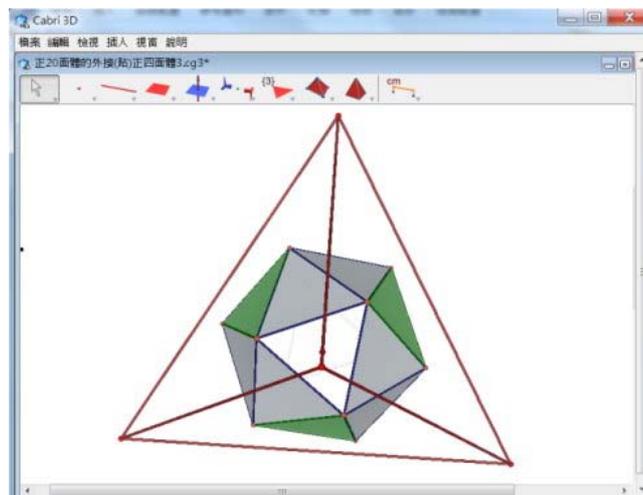


Fig 1 Cabri3D working environment

Cabri3D, with its dynamic manipulation environments, has three important attributes. First, students can directly manipulate mathematical objects represented on the screen. Second, mathematical objects stay coherent at all times as they are dragged. Third, students feel that they are involved with the objects they are manipulating, that is, they are immersed in the environment.

For students, it is designed to help explore and understand concepts in mathematics. Students can develop their algebraic equation solving skills through playing Cabri3D and print out (See Fig 2). As shapes and positions change, all mathematical relationships are preserved, allowing teacher and students to examine an entire set of similar cases in a matter of seconds.

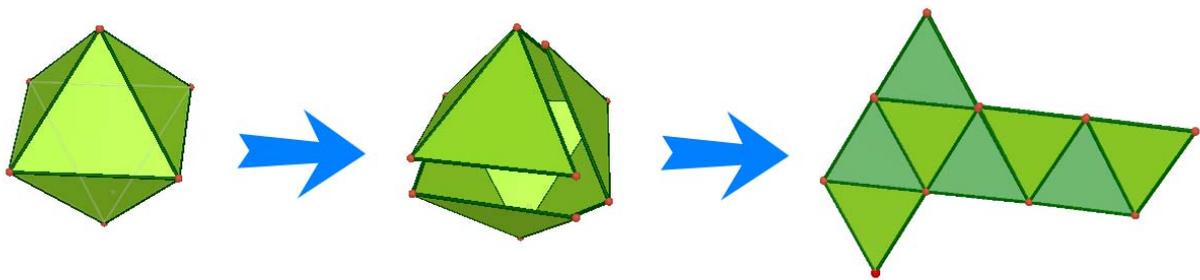


Fig 2 spread out the solid

Teaching Strategy

Our research plan is based on the theories of Van Hiele and Vygotsky. Theoretical foundations such as constructivism, social interaction and discussions, the use of smartboard, iPads, Cabri3D, cooperative learning groups, and higher order thinking are just a few of the theories that support our teaching philosophy and this research.

We proposed a SIC (smartboard, iPad, Cabri3D) teaching model which is a blend of classroom materials, self-paced e-learning and assessments. A broader extension of this model is to do more than facilitate performance, by actually promoting learning as well. As shown in Fig 3, considerable initial and ongoing training and professional development has been provided. The iPad can support classroom instruction and performance support. It can display HD video. It has a video out via the dock connector, so a teacher can display keynote presentations from the iPad alone. E-learning on iPad consolidates the delivery of these materials into a single tablet platform and leverages the best the iPad has to offer: ease of distribution, powerful e-reader functionality, rich multi-media and unparalleled navigability.

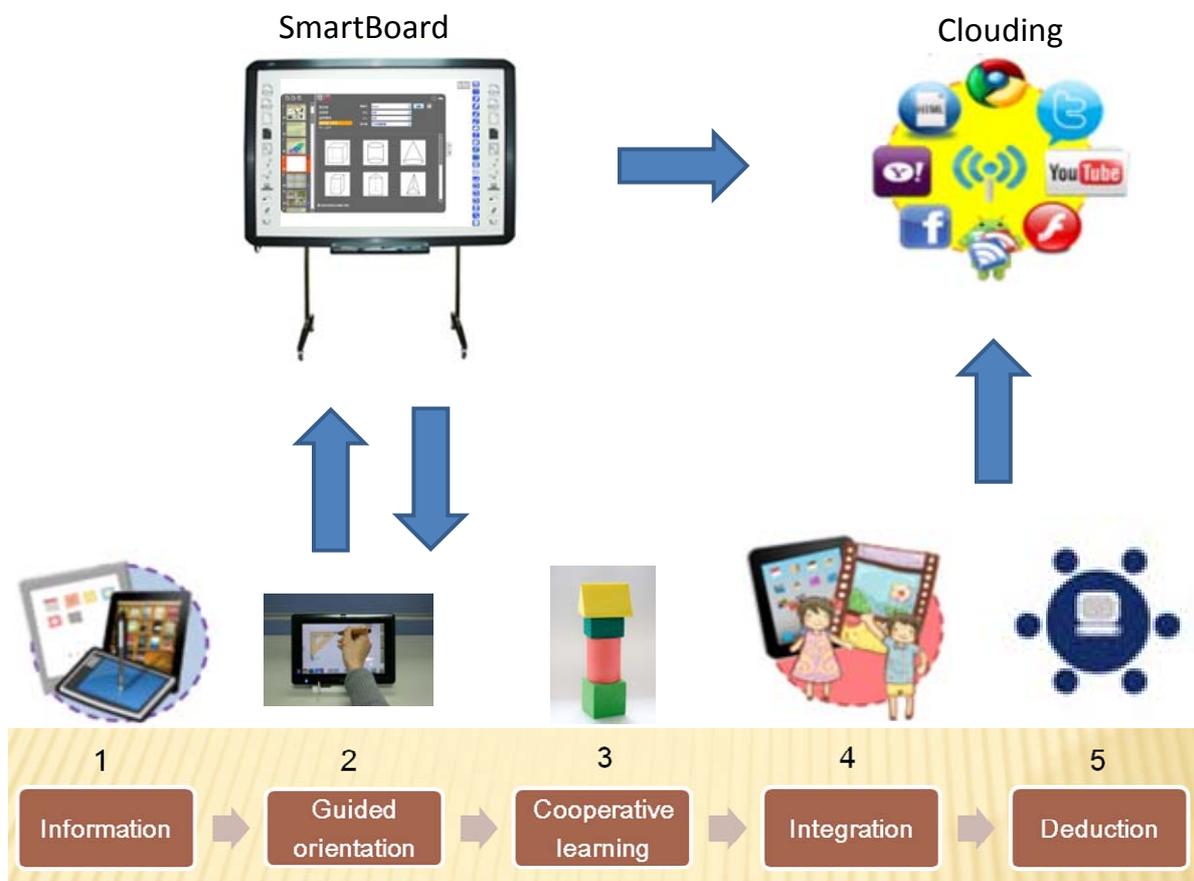


Fig 3 SIC teaching model

The teaching procedure includes five steps as follows.

1. Information

This activity explored and manipulate shapes help students develop a sense of spatial relationships. The use of the smartboard in combination with an iPad in the teacher’s hand provides a mobile platform from which classroom activities can be initiated. Ask students to classify and search for examples of different shapes in their environment, and allow students to participate from their own seats in a variety of interactive activities. The iPad lets students organize their concept better as well as create their own solid categorization from the classroom as well as internet reference sources. The tasks included drawing shapes, identifying and defining shapes, sorting shapes, determining a mystery shape, establishing properties of parallelograms, and comparing shapes explicitly by means of properties of their components. Descriptions of types of shapes by explicit use of their properties, rather than by type names, even if known. Finally, teacher help students write the names of the objects on the sheet entitled short. Thus, this activity lets students have the ability to sort shapes according to a variety of mathematically precise attributes and to complete definitions of types of shapes. Once students have finished, attach the categorization of the corresponding list of objects, upload the diagram to website for sharing (See Fig 4).

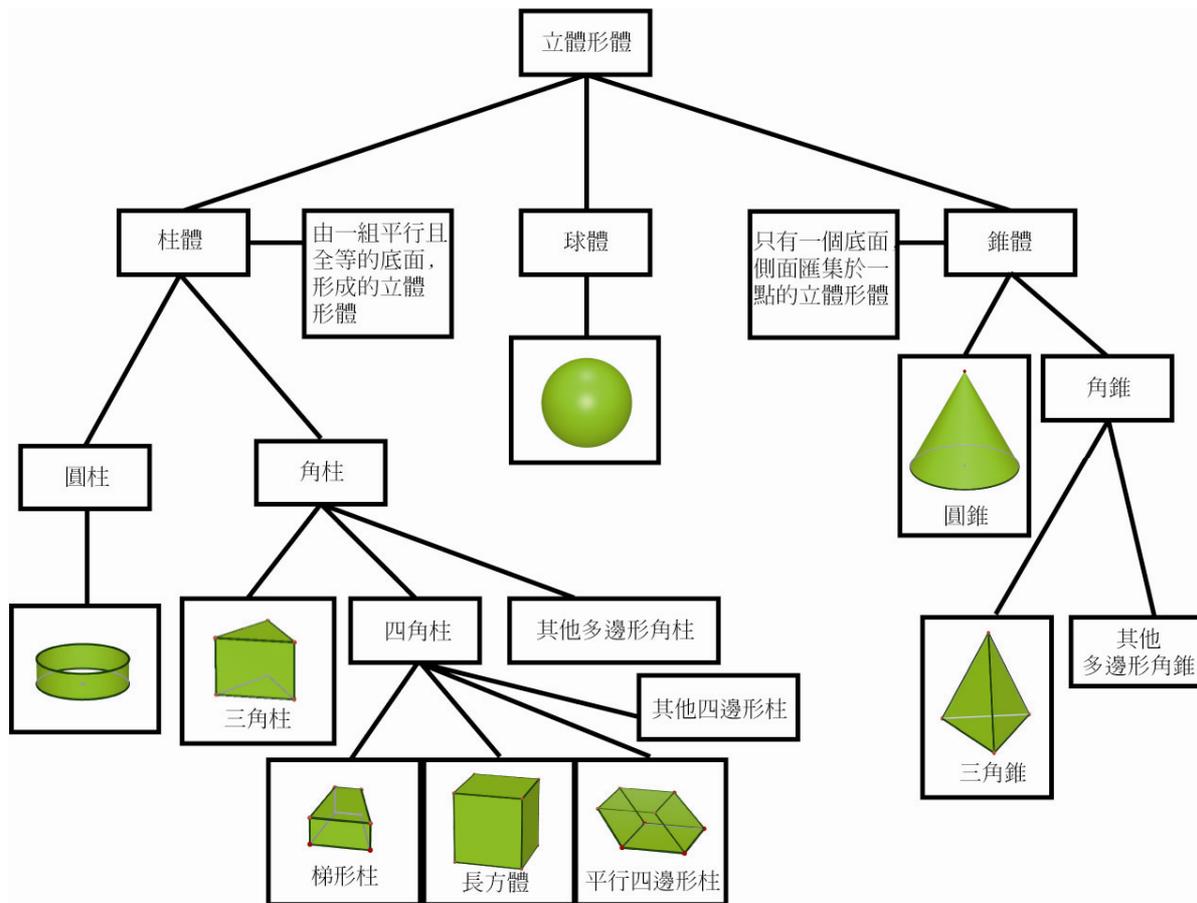


Fig 4 solid geometry classification

2. Guided orientation

This activity encourages students’ incorporation of conventional terminology by consistently and accurately referring to objects by their mathematical names, for example, cube, cylinder, or rectangular prism. Begin by showing students Cabri3D with smartboard, iPad and projector. The teacher provides students conceptual material and examples beforehand, reactivate relevant knowledge to meet their unique learning needs. Students develop spatial visualization and reasoning abilities as they predict and then encourage students to be precise in their iPad movements. Ask students to explain their thinking and ask students if they can think of situations in which the size of the forward step would make a difference. As students become better at navigating, increase the amount of solids, e.g., spheroid, cone, pyramid. Give students an opportunity to check their estimates by measuring each of the objects. Challenge student to measure volume as many different shapes as they can using exacting formulas. The students’ behavior on the tasks was consistent with the ability to modify definitions and immediately accept and use definitions of new concepts. They explore various ways of getting a volume by marking lengths, comparing lengths, and help students develop ideas of distance, direction, and relative position

in space. In this activity, students identify and describe different lengths within the iPad and on a figure, and follow directions to measure volume (See Fig 5).

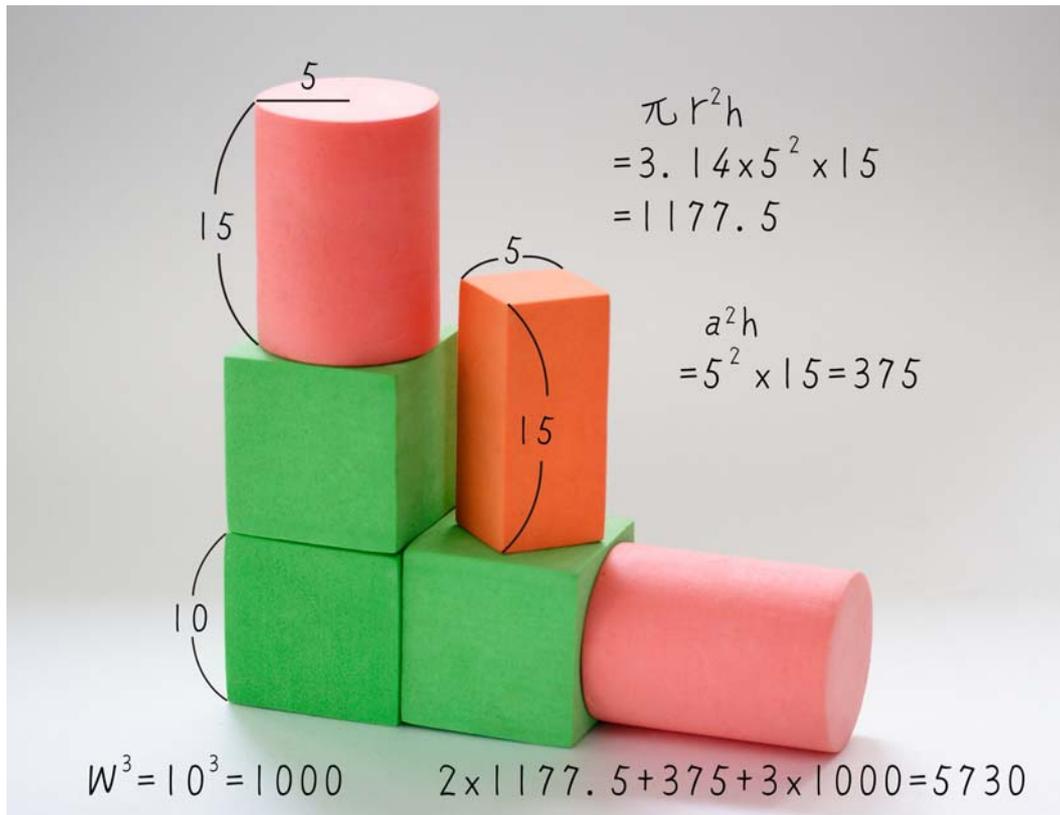


Fig 5 student use iPad and share others the computing

3. Cooperative learning

The teacher offers a range of exercises to ensure students comprehend the content and self-progressing that uses social techniques to encourage achievement. A set of solids was spread out on the table and being put together that are alike in a different way. Encourage students to look for familiar shapes in the geometric quilt designs. Invite them to pick out their favorite design and say why they like it. The students were asked how the figures differed and how many different solids he or she could draw. The learning space is no longer fixed but based on connecting student with each other and information through collaborative spaces and communities. This task investigated the properties that students varied to make different solids and explore the effects of transformations in different geometric solids. They explore different new shapes that can be made by combining solids and explore what happens when shapes are cut or divided into smaller shapes. Students use spatial reasoning as they indicated they had enough clues to decide the shape, they were asked how they knew with certainty and whether another clue would change their minds. This cooperative learning was continued as long as the student could come up with new sorting properties. Then they were asked to give an example of each term with which they were familiar (See Fig 6).



Fig 6 students design a shape of compound solids

4. Integration

This activity elicited formal inference and addressed the role of necessary versus sufficient conditions to determine a shape. The students were asked to draw a shape that was different from the other group’s design, and so forth as long as the question proved fruitful. Have students described their pictures in terms of the shapes they have created. On the identifying and defining activity, the students included many additional shapes. Some students identified the shapes completely correctly and defined them by properties of their components, perhaps including some redundancies. Encourage students to explore what happens when they divide other shapes such as a spheroid, cone, or pyramid to make new shapes. Students can collaborative create new understandings that may only be personally new, or indeed may be new contributions to shared understandings. The number of knowledge nodes is absolutely immense and do have social aspects that encourage discussing and learning with classmates. Students are focused on delivering a solid geometry experience while teaching concepts and information along the way. The iPad presents content created with Cabri3D in accordance with personal preference by the students (See Fig 7).

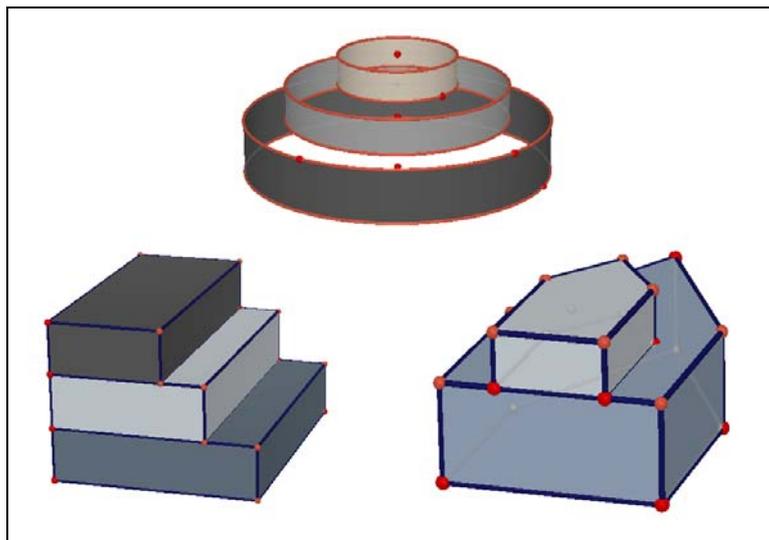


Fig 7 co-creation with Cabri3D

5. Deduction

Encourage students to communicate mathematically by talking about the shapes they see or have constructed in their geometric designs. They identify and describe geometric shapes in the designs and analyze how shapes are transformed to create the overall pattern (See Fig 8). Since custom polyhedron is rarely designed for public use,

the others have to say how much the volume is. When they have finished, have them challenge someone else to measure volume. Students could also have the device capture performance, and share it with classmates for feedback. They point to additional information about their creation online, such as idea, character, and process. Finally, the teacher rewards students for reaching specific goals and targets.

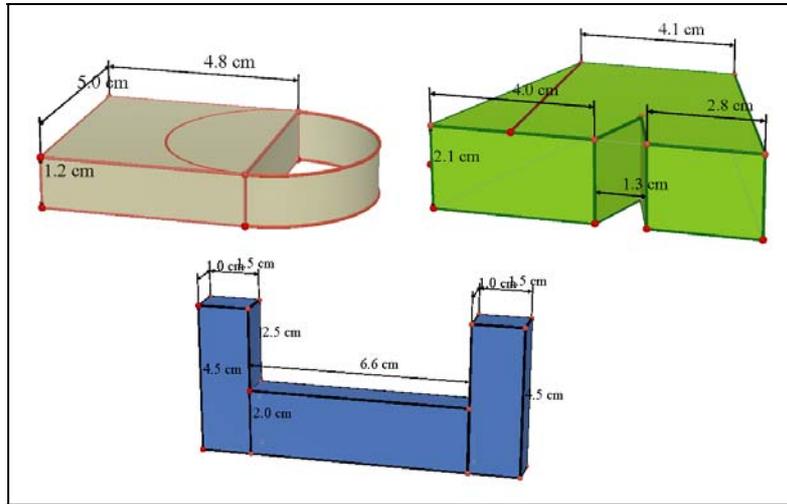


Fig 8 solids designed for competition

Sample

The samples were all second grade students enrolled at a Taipei high school. Treatment group has 158 students which have gathered from 4 classes. All students sit in their cozy locations in front of an iPad. The control group has 158 students which have gathered from 4 classes where tech is classroom based. Students in the control group used their student textbooks for instructional purposes, and used corresponding worksheets for practice.

According to math level, every class was divided into three parts by 27% and 73%. The top 27% of the class is high grade level. The bottom 27 % of the class is low grade level. And the rest of the class is middle grade level. As shown in Figure 1, in the treatment group, the high grade number of students is 44, the middle grade number of students is 70, and the low grade number of students is 44. In the control group, the high grade number of students is 44, the middle grade number of students is 70, and the low grade number of students is 44.

Table 1 Number of sample data

	Treatment group					Control group		
Class	1	2	3	4	5	6	7	8
High	11	11	11	11	11	11	11	11
Middle	18	17	18	17	17	18	17	18
Low	11	11	11	11	11	11	11	11

The students were assigned for the duration of the geometry unit, which lasted for three weeks instructional days.

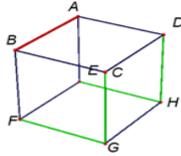
Evaluation

Over three weeks, we ran an experiment asking students to explore by way of cooperating learning gradually with their iPads that they use in a solid geometry-related context. Clearly the students know what the device is capable of and are keen to exploit that functionality.

When the experiment is over, we had a final examination to measure the geometric abilities of students as a function of van Hiele level. One month later, we had a posttest to investigate the effects of instruction on a student's predominant van Hiele level. Both the two examinations include five parts, each part has four questions, i.e., ‘relations between lines’ (See Fig 9), ‘relations between line and plane’ (See Fig 10), ‘relations between planes’ (See Fig 11), ‘theorem of three perpendiculars’ (See Fig 12), ‘combined concept’ (See Fig 13).

例題2
正方體的12個邊中共有多少對歪斜線？

□ 答：



Hint:
與每一邊互相歪斜的稜線計有4對

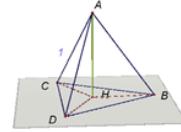
以AB邊為例，共有 4 對歪斜線，
因正立方體有12個邊，所以共有 $12 \times 4 = 48$ (總計)
因每一對各算了2次，故共有 $48 \div 2 = 24$ (對)

Fig 9 relations between lines

例題3

下圖是邊長為1的正四面體A-BCD，從頂點A對底面BCD做垂線AH交底面於H點，試求正四面體的高AH。

Hint:
直觀上，
垂足H是正△BCD的重心，
但理由何在？



證明 \overline{AH} 與底面BCD垂直

$\Rightarrow \overline{AH}$ 與 \overline{BH} 、 \overline{CH} 、 \overline{DH} 皆垂直

又A-BCD為正四面體 $\Rightarrow \overline{AB} = \overline{AC} = \overline{AD}$

由畢氏定理： $\overline{BH} = \overline{CH} = \overline{DH} = \sqrt{1^2 - \overline{AH}^2}$

$\therefore H$ 為正△BCD的外心，也是其重心

Fig 10 relations between line and plane

例題4

如下正四面體A-BCD中，若 $\overline{AB}=1$ ，且E為 \overline{CD} 的中點

(1) 試求 \overline{AE} 與 \overline{BE} 的長度

(2) 若兩半平面CDA與CDB的夾角為 θ ，試求 $\cos\theta$ 。

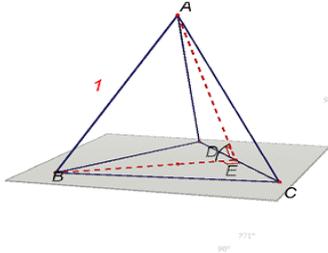


Fig 11 relations between planes

三垂線定理敘述及證明

□ 設 $\overline{PA} \perp$ 平面E於A，L為平面E上不通過A的直線

(1) 若由A點向L作垂線，其垂足為B，($\overline{AB} \perp L$ 於B)

則 $\overline{PB} \perp L$ 。

證明：

在L上任取異於B的C點

連接 \overline{PC} 和 \overline{AC}

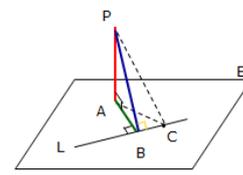
由畢氏定理

$$\overline{PC}^2 = \overline{PA}^2 + \overline{AC}^2$$

$$= \overline{PA}^2 + (\overline{AB}^2 + \overline{BC}^2)$$

$$= (\overline{PA}^2 + \overline{AB}^2) + \overline{BC}^2$$

$$= \overline{PB}^2 + \overline{BC}^2 \Rightarrow \overline{PB} \perp L$$



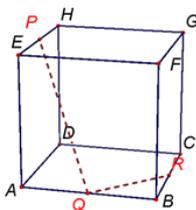
(2) 反之，若 $\overline{PB} \perp L$ 於B

則 $\overline{AB} \perp L$ 於B

Fig 12 theorem of three perpendiculars

例題5

如圖，ABCD-EFGH為一正方體，P、Q、R分別為EH、 \overline{AB} 、 \overline{BC} 的中點，試求 $\angle PQR = ?$



直觀上， $\angle PQR$ 應該是 90° ，但是該如何說明呢？

Fig 13 combined concept

Statistics

This study is interested in the effect of teaching and level of achievement. Group descriptive statistics, such as mean and standard deviation, were calculated to classify and summarize data. For the comparisons between different teaching and practice activities, Two-way ANOVA with $\alpha = 0.01$ were conducted.

Two-way ANOVA with unequal number of observations per cell was performed to analyze the data on measurement of Teaching and Level, of each parameter of different group of subjects. The Teaching factor depends on the teaching model use iPad or not. The Level factor includes high grade level, middle grade level, and low grade level. This study does not concern the effect of different grade levels. We prefer to know the interference degree cause by Level. All tests were two-tailed and $p < 0.01$ was considered statistically significant. However, the data was not equal across groups. When the sample sizes within the levels of our independent variables are not equal, we have to handle our ANOVA differently than in the typical two-way case. In our study, three (high, middle, low) grade level students participated in the iPad environment. As such, we

should take action to compensate for the unequal sample sizes in order to retain the validity of our analysis.

According to Myers (1979), when the group sizes are sharply unequal (largest/smallest > 2) and a statistical test shows that the population variances are unequal. The ratio of treatment group sizes to control group size in this study was $70/44 = 1.59$ (less than 2). This indicated that the F statistic was robust.

The success of the structured interview, using a specific script as a basis, enabled the teachers to compare many students' responses to the same tasks. Tasks that involved a variety of environments in which the concepts were embodied (drawing, identifying from pictures, sorting, and solving abstract problems) revealed modes of reasoning about specific concepts that the teachers could identify with confidence.

DATA COLLECTION AND ANALYSIS

1. On the high grade level the treatment group had a mean of 18.3 out of 44 items, with a standard deviation of 1.07. The control group had a mean of 18 with a standard deviation of 1.26. On the middle grade level the treatment group had a mean of 15 out of 70 items, with a standard deviation of 1.37. The control group had a mean of 13.4 with a standard deviation of 2.02. On the low grade level the treatment group had a mean of 9.6 out of 44 items, with a standard deviation of 6.9. The control group had a mean of 18 with a standard deviation of 2.53.

An unweighted mean is calculated by taking the average of the individual group means. Thus, we can derive our unweighted means by summing the means of each level of our independent variables and dividing by the total number of levels. The harmonic mean of n is derived 50.21 (n11 = 44, n12 = 70, n13 = 44, n21 = 44, n22 = 70, n23 = 44).

As shown in table 2, with $\alpha = 0.01$ (99% confidence), the ANOVA results indicated a significant main effect for the Teaching factor, $F(1, 310) = 46.09$ ($p < 0.01$), a significant main effect for the Level factor, $F(2, 310) = 649.22$ ($p < 0.01$), as well as the interaction between them, $F(2, 310) = 9.43$ ($p < 0.01$). F values for all three level groups were found to be highly significant ($p < 0.01$).

Table 2 Results of ANOVA for the Teaching and Level data

Variance resource	SS	df	MS	F	Significance
A (Teaching)	177.10	1	177.10	46.09*	$p = 0.0000$
B (Level)	4988.76	2	2494.38	649.22*	$p = 0.0000$
A × B (Interaction effect)	72.48	2	36.24	9.43*	$p = 0.0032$
w.cell(error)	1191.05	310	3.84		

Another way to say this is that we don't want the Teaching variable contaminated by the variance it shares with Level: we want to know what the effect of Teaching is holding a Level constant. As shown in table 3, the Teaching effect in middle grade level and in low grade level was found to be statistically significant ($p < 0.01$). But the Teaching effect in high grade level was not significant ($p > 0.05$).

Table 3 Level group-wise analysis of ANOVA

Variance resource	SS	df	MS	F	Significance
A (Teaching)					
in B1 (high grade level)	1.38	1	1.38	0.36	$p = 0.5214$
in B2 (middle grade level)	89.60	1	89.60	23.32*	$p = 0.0072$
in B3 (low grade level)	158.23	1	158.23	41.18*	$p = 0.0006$
w.cell(error)	1191.05	310	3.84		

2. A 2x2 (final-posttest by control-treatment) mixed model ANOVA with $\alpha = 0.01$ was conducted to examine reservation from final to posttest. The posttest is held one month later since final test. The results of the ANOVA indicated a significant main effect for the within factor, $F = 4.8132$, $p = 0.0032$ (< 0.01).

3. We wonder if changing a teaching formula, process or material might deliver a better learning effect depend on five teaching units. Use one-way ANOVA to determine if there's a statistically significant difference between two alternatives.

As shown in table 4, both the p -value of unit 1: relation between lines and the p -value of unit 2: relation between line and plane are greater than the significance level (0.01), so we cannot reject the null hypothesis that the means are equivalent. Each p -value of the rest three units are less than the significance level (0.01), so we can reject the null hypothesis and safely assume that SIC teaching model affects learning effect.

Table 4 Results of ANOVA for five teaching units

Unit	F	p
1. relations between lines	0.1044	0.7543
2. relations between line and plane	1.7238	0.2416
3. relations between planes	11.0224	0.0024**
4. theorem of three perpendiculars	8.6046	0.0043**
5. combined concept	9.0027	0.0039**

4. The accumulated data of the questionnaire.

Table 5 Summary of questionnaire from 316 students

Question	Strongly disagree	Disagree	No comment	Agree	Strongly agree
Q1. The curriculum matched the life.	0%	6%	15%	35%	44%
Q2. Teaching can promote my learning emotion.	4%	14%	19%	32%	31%
Q3. The teacher can present the course content clearly.	0%	12%	22%	33%	33%
Q4. The teacher can use each equipment properly.	0%	2%	4%	51%	43%
Q5. Cabri3D helps and reinforce my learning.	4%	8%	12%	30%	46%
Q6. iPad helps and reinforce my learning.	0%	10%	12%	26%	52%
Q7. I am making better progress with the iPad than I did without it.	0%	0%	2%	17%	81%
Q8. My achievement has improved since the iPad was introduced.	0%	8%	6%	31%	55%

5. The comments that were listed most often, and include some feedback from teachers regarding their observed students in class.

A1. iPad has the flexibility to meet teaching needs regardless of subject matter, technological expertise, grade level, or curriculum.

A2. iPad provides a faster, more dynamic and engaging way to demonstrate mathematical concepts than drawing on the board.

A3. Concepts that students frequently find difficult become very clear when they see visual representations on the screen and interact with them using iPads. Students using iPads in the classroom felt better prepared with their homework and that the Cabri3D assisted them with their geometry taking skills.

A4. There is no camera, so the virtual classroom is likely to be an audio-only experience. Because the iPad does incorporate a microphone and speakers, learners should be able to conference via Skype or Google Voice.

A5. The large capacitive screen of the iPads allows more than one person to view and interact with the device without passing it around. Concepts that students frequently find difficult become very clear when they see visual representations on the screen and interact with them using iPad.

A6. It is clearly surprising that some students show such an improvement in their ability to recognize representations of theorems in more complex diagrams, when they could not recognize the same representations in simpler diagrams.

A7. In that way of teamwork and competition, students drive activity and advancement in ways that generate rewards and motivation.

A8. Using smartboard as well as iPads in share activities as well as group work, students will be more likely to develop a critical opinion.

DISCUSSION

1. Apart from this drawback, Cabri3D along with iPad, seem to be a great tool to use in the classroom. Use in 'relation between planes', 'theorem of three perpendiculars', 'combined concept' identified significant learning gains compared to 'relation between lines', 'relation between line and plane'. However, there does appear to be a relationship between level of use and the ability of math. Overwhelmingly students, 78% (Q6), believe that the applications and freedom provided by the iPad and the Cabri 3D combination is a much better solution for their learning needs. Use of the iPads facilitated and encouraged group collaboration that it had a positive impact on achievement. The use of the iPads is becoming firmly embedded. A significant number of students, 98% (Q7), used the iPad in the majority of their lessons.

2. With applications available today, students have the capabilities to connect to the smartboard from their own iPad in order to share and present their own work. Thus geometry resources, science videos, online maps, can be referred to instantly and used to extend learning in ways simply not possible otherwise. The use of mind mapping is particularly important since it supports the development of higher level thinking skills and better analysis of information and connectivity of ideas and events. A lesson observation where the technique was used to reinforce understanding demonstrated that given a choice most students chose to use iPad rather than pencil and paper, the facility for rapid edits and the ability to readily share their mind map as important. A significant majority of students (86%, Q8) felt they worked better with an iPad rather than pen and paper, found Cabri3D beneficial to learning and wanted to make greater use of the devices. The technique was used to reinforce understanding demonstrated that given a solid geometric concept most students chose to use figure representation rather than algebra equation, the facility for rapid edits and the ability to readily share their mind map as important. For high grade level students, since there was not a significant difference between the experiment group and the control group students on measures of performance when experiencing the different instructional environment, does it follow that students should always coexist in SIC classes with their iPad peers? This research has demonstrated that a computer-based, dynamic instructional environment can provide for successful outcomes in students' solid geometry learning. It remains to be shown that similar instructional environments can afford students the opportunity to construct other areas of mathematics.

3. During the course of the study, several features of the levels emerged that we were not aware of initially. As van Hiele has suggested, neither person could understand the other's reasoning, resulting in frustration and discouragement. Therefore, it is very significant in the learning process to recognize and assess a student's intellectual capacity. First, the levels appear to be complex structures involving the development of both concepts and reasoning processes applicable to many task environments. Although the van Hiele have theorized that the levels are discrete structures, this study did not detect that feature. The occasional difficulties that teachers had in deciding between levels while making level assignments can be considered as evidence questioning the discrete nature of the van Hiele levels. Some students exhibited different preferred van Hiele levels of reasoning on different tasks. It is interesting to notice how these data agree with the constructivist learning theory (Hmelo-Silver, Duncan, & Chinn, 2007). Each student brings different experiences to the learning situation and hence may be more capable on different aspects of the same problem. As van Hiele has suggested, neither person could understand the other's reasoning, resulting in frustration and discouragement. Some even oscillated from one level to another on the same task under probing by the interviewer. Thus, the levels appear to be dynamic rather than static and of a more continuous nature than their discrete descriptions would lead one to believe. Students may move back and forth between levels quite a few times while they are in transition from one level to the next.

4. How these students might reason about shapes in a formal way was most unclear. Concept formation in geometry may well occur over long periods of time and require specific instruction. Most formal learning provides artificial contexts, but a closer approximation to the real world is through virtual worlds that add the immersion for students would experience in the real world. Memorization may be their only recourse. Students in the study who appeared to reason at the level of potential development used different problem-solving processes under teacher guidance or in collaboration with more capable peers. This phenomenon would also occur between a teacher and a student who are operating at different levels. It appears that tutoring is so effective because it maximizes the Zone of Proximal Development (Vygotsky, 1978). In the classroom, this concept is connected with another instructional design concept of scaffolding. The concept of scaffolding is a process, with each failures, a teacher provides with minimal feedback and then vary the response that will enable a student to build on prior knowledge. And once the stage of zone of proximal development has been reached, the specific guidance is gradually removed. This encourages the student to work and internalize new concepts. Students working in collaboration to design shapes may be seen as an elevation to higher ground what Vygotsky (1978) called "second-order symbolism, which involves the creation of written signs for the spoken symbols of words". The aids are in the form of verbal and nonverbal communication and model behavior. Finally, the notion of a

SIC teaching model that identifies deep concepts and structures of mathematics makes it possible to elevate student's learning of solid geometry to higher ground (Bruner, 1985), so that the new higher concepts in turn transform the meaning of the lower (Vygotsky, 1986). It is therefore paramount that activities are designed to allow such social interactions to take place.

5. The SIC teaching model could help students learn from these principles and put them into practice. This iPad presentation tool is easy to use, letting students annotate, animate, and narrate explanations and presentations. For iPad to learn and review their solid geometry-solving skills, it is addicting and a fun way to get students to learn. Think more deeply about a learning approaches that integrates the iPad into the instructional environment, teachers and students might discover many new ways in which the device can expand and enhance the learning environment. Ultimately, this type of Cabri3D software could increase competition among classmates and offer creative opportunities for the fertile minds of innovative students. This further supports the idea that attitude is a factor that is open to influence (Volman & van Eck, 2001). Think more deeply about a learning approaches that integrates the iPad into the instructional environment, teachers and students might discover many new ways in which the device can expand and enhance the learning environment.

CONCLUSIONS

1. The Cabri3D, in conjunction with the use of the iPad and smartboard, offers new possibilities for our teaching professionals.

These essential components of the phase-based instructional environment using iPad helped improve students' van Hiele levels of geometric thinking about solids. This suggests for this sample that with well-designed instructional activities, appropriate tools, and teacher guidance, students can learn important solid geometric concepts with increasing understanding. By directly manipulating the Cabri3D to generate many examples of solids, the students were able to recognize its shape and understand that solids by analyzing the measurement of its volumes. Through their dynamic manipulation and reflecting on those actions, students were able to understand properties of solids. Teachers need to use technology appropriately based on students' van Hiele levels to avoid mismatches between levels. It would be appropriate to investigate students' responses on tasks involving other geometry concepts, such as measurement, transformations, congruence, and similarity.

2. Knowledge alone cannot become developed, but it has to be a channel through which intellectual stimulation and development occurs.

This concept along with more modifications and changes has played a significant role in the way education has been imparted. Optimizing intellectual capacity that could surpass an instructor is what zone of proximal development aims for. The idea being that an association, if not an immediate then a gradual one has to be built between concepts, experiences and reactions. It is a more challenging task to find ways in which to determine what makes a representation fall within a student's direct match for their current level of ability, or just a bit beyond. If the challenge is too hard, then a student would become overwhelmed and stressed out by the task. If it was too easy, then this same student would be bored. Teachers need to know their students' levels of geometric thinking and the content areas they are teaching, and also have adequate resources to support their work so that they can serve in the various roles competently throughout all the five phases of learning.

3. Teachers have to identify a suite of social-learning skills and teaching styles to develop that is relevant to support the needs of specific classes and students.

The iPad enables a host of activities such as referencing, collaborating, and creating content. Our SIC teaching model includes not only providing the infrastructure, tools, and knowledge, but also developing learners as learners. The students developed their ability to perform, not just their knowledge. E-learning resources don't replace the classroom experience, but they do provide an extremely wide range of learning resources that teachers and students can take advantage to support classroom learning as well as to develop individual pathways based on actual student need. Teachers need to organize sequences of lessons comprising well designed instructional activities that move very deeply through the levels of geometric thinking and the five phases of learning, not only to enrich students' thinking at the current level but also to move them toward the next level in order to develop a deeper understanding of the concepts.

4. Encourage students to use iPad across the mathematics curriculum to cover the insufficient of thinking.

If students are not familiar with the iPad, they might want to look into life experience with peer. Characterize contexts both geographic and semantic, so that both the type of event students is engaged in as well as where they are and what would be useful to them here and now. The teacher should have the ability to truly create new learning experiences as well as tailor content to their classes, articulating them, assessing them, and developing them. Teacher in the future will be adapting the study's procedures to investigate other geometric concepts and linking individual learning goals, social learning power, distributed cognition, and spaced practice in a powerful,

long-term learning opportunity. The capability for sharing this space with others becomes powerful for shared learning, with reciprocal performing and critiquing to co-develop ability.

REFERENCES

- Banchoff, T. F. (1990). Dimension. In L. A. Steen (ed.). *On the shoulders of giants: New approaches to numeracy*. Washington, DC: National Academy Press, 11–59.
- Battista, M. T. (1994). A Forum for Researchers. *Journal for Research in Mathematics Education*, 25(1), 86-99.
- Bruner, J. S. (1985). Vygotsky: A historical and conceptual perspective. In J.V.Wertsch (Ed.), *Culture, communication and cognition: Vygotskian perspectives*. Cambridge: Cambridge University Press.
- Carroll, W. (1998). Geometric knowledge of middle school students in a reform-based mathematics curriculum. *School Science and Mathematics*, 98(4), 188-197.
- Chang, K. E., Sung, Y. T. & Lin, S. Y. (2007). Developing geometry thinking through multimedia learning activities. *Computers in Human Behavior*, 23, 2212-2229.
- Christou, C., Jones, K., Pitta-Pantazi, D., Pittalis, M., Mousoulides, N., Matos, J. F., Sendova, E., Zachariades, T. & Boytchev, P. (2007). Developing student spatial ability with 3D software applications. Paper presented at the 5th Congress of the European Society for Research in Mathematics Education (CERME5), Larnaca, Cyprus, 22-26.
- Clements, D. H. & Battista, M. T. (1993). Geometry and Spatial Reasoning. In Grouws, D.A.. *Handbook of Research on Mathematics Teaching and Learning*. 420-427.
- Clements, D. H., Battista, M. T., Samara, J. & Swaminathan, S. (1997). Development of students' spatial thinking in a unit on geometric motions and area. *The Elementary School Journal*, 98(2). 171-186.
- Corley, T. L. (1990). *Students' Levels of Thinking as Related to Achievement in Geometry*. Unpublished Doctoral dissertation, Arizona State University, Tempe.
- Crowley, M. L. (1987). The Van Hiele Model of the Development of Geometric Thought. In M. M. Lindquist & A. P. Shulte(Eds.), *Learning and Teaching Geometry*, K-12, 1-16.
- Flores, A. (1993). Pythagoras Meets Van Hiele. *School Science and Mathematics*. 93(3), 152-157.
- Hmelo-Silver, C. E., Duncan, R. G., & Chinn, C. A. (2007). Scaffolding and Achievement in Problem-Based and Inquiry Learning: A Response to Kirschner, Sweller, and Clark (2006). *Educational Psychologist*, 42(2), 99–107.
- Hutchins, E. (1996). *Cognition in the Wild*. Boston: MIT Press.
- Hwang, G. J., Chu, H. C., Lin, Y. S. & Tsai, C. C. (2011). A knowledge acquisition approach to developing Mindtools for organizing and sharing differentiating knowledge in a ubiquitous learning environment. *Computers & Education*, 57(1), 1368-1377.
- Hwang, G. J., Wu, P. H. & Ke, H. R. (2011). An interactive concept map approach to supporting mobile learning activities for natural science courses. *Computers & Education*, 57(4), 2272-2280.
- Hwang, G. J., Shi, Y. R. & Chu, H. C. (2011). A concept map approach to developing collaborative Mindtools for context-aware ubiquitous learning. *British Journal of Educational Technology*, 42(5), 778–789.
- Mason, J. (2003). Structure of attention in the learning of mathematics, in J. Novotná (Ed.), *Proceedings, international symposium on elementary mathematics teaching* (pp. 9-16). Prague: Charles University.
- Melhuish, K. & Falloon, G. (2010). Looking to the future: M-learning with the iPad. *Computers in New Zealand Schools: Learning, Leading, Technology*, 22(3).
- Myers, J. (1979). *Fundamentals of experimental design*, 3rd ed., Boston: Allyn & Bacon.
- Perdikaris, S.(1996). A System Framework for Fuzzy Sets in the van Hiele Level Theory of Geometric Reasoning. *International Journal of Mathematics Education in Science and Technology*.27(2), 273-278.
- Piaget, J. (1959). *The language and thought of the child*, 3rd ed., Routledge and Kegan Paul, London
- Rogoff, B. (1990). *Apprenticeship in Thinking: Cognitive Development in Social Context*. New York. 14-16.
- Sabean, M. P. & Bavaria, R. (2005). *Sylvan Learning Center math research*. Sylvan Learning, Inc.
- Schwartz, J. & Yerushalmy, M. (1992). *The Geometric SuperSupposer*. Pleasantville, NY: Sunburst Communications.
- Senechal, M. (1990). Shape. In L. A. Steen (ed.). *On the shoulders of giants: New approaches to numeracy*, Washington, DC: National Academy Press, 139-181.
- Sharp J. M. & Hoiberg K. B. (2001). And then There was Luke: the Geometric thinking of a young mathematician. *Teaching Children Mathematics*, 7(7), 432.
- Van Hiele-Geldof, D. (1957). *De didaktiek van de meetkunde in de eerste klas van het V.H.M.O*. Unpublished doctoral dissertation, University of Utrecht.
- Van Hiele. (1986). *Structure and insight: A theory of mathematics education*. Orlando, FL: Academic Press.
- Van Hiele–Geldof, D. (1984). *The didactics of geometry in the lowest class of secondary school*. In D. Fuys , D Geddes & R. Tischler (Eds): english translation of selected writings of Dina van Heile-Geldof and Pierre M. van Heile, 1 – 214). Brooklyn, NY: Brooklyn College.

- Volman, M. & van Eck, E. (2001). Gender equity and information technology in education: The second decade. *Review of educational research*, 71(4), 613-634.
- Vygotsky, L. S. (1978). *Mind and society: The development of higher psychological processes*. Cambridge, MA: Harvard University Press.
- Vygotsky, L. S. (1986). *Thought and Language*. Cambridge, MA: Harvard University Press.
- Wilson, M. (1990). Measuring a van Hiele Geometry Sequence: A Reanalysis. *Journal for Research in Mathematics Education*. 31(3), 230-237.