Mathematics and Language

Assist.Prof.Dr.Mehmet Çağlar
Eastern Mediterranean University
Famagusta, North Cyprus
mehmet.caglar@emu.edu.tr

1. Introduction

If we are to say that mathematics deals only with numbers, then the assumption that having mathematics without language will undoubtedly be correct. To some extent this assumption is true but the main point that needs attention at this stage is how would it be ever possible to talk about mathematics without language.

Language is the means of communicating, expressing, and interpreting information and ideas. The modern era demands literacy. Not only learners, i.e., the students, but even those at the workplace should be able to translate between representations, within mathematics, between mathematics and other areas as well; to communicate findings orally and in writing. The connections between mathematics and other disciplines are quite obvious. Mathematics gives people the power and utility to express, understand and solve problems in diverse settings (NCTM, 1985).

According to Libby Krussel (1996) since mathematics has words and symbols which is an extension of existing language and since it has its own syntax and grammar, it is possible to say that math is a language.

As it is quite rightfully claimed that math is a language, then it may as well be asserted that math is learned just as a language is. Children learn using telegraphic language implying that they start off with one or two word utterances, and then they pass on to formulaic speech which is the usage of short structures. The same process applies to learning mathematical concepts as well, like in learning addition first and then multiplication. Otherwise, the latter will not signify much to the learner.

2. What is Language?

Albert Einstein (1938) stated at one point that "most of the fundamental ideas of science are essentially simple, and may, as a rule, be expressed in a language comprehensible to every one." What is this language Einstein is talking about? Before we delve any further into the main scope of our topic, it would be proper to define the term *language.* Language is a means of communication of complex ideas through the use of arbitrary symbols to encode these ideas.

Among many of the functions of language, communicating our ideas ranks first. When we want to talk about ourselves, convey information, ask for information, exchange facts and opinions, the only option we have is to use language. The use of language in this manner is often referred to as "referential" where the communication of ideas is at times a marginal or an irrelevant consideration according to some language scholars (Crystal 1995)

Language as "emotive" or "expressive" function consisting of conventional words of phrases or semi-linguistic sounds usually referred to as "interjections." Language is also used for maintaining rapport between people, which the anthropologist Bronislaw Malinowski (in Crystal:10) calls as "phatic communion" implying the social function of language.

The most common use of language as an instrument of thought is, according to Crystal (1995), when people perform mathematical calculations in their head. This mental act is sometimes uttered verbally, however, language used in this way need not always be spoken aloud or written down. The Russian Psychologist Vygotsky (in Crystal: 13) argued that to evoke a sequence of thoughts language is essential.

Certain expressions need to be used in order to carry out communication with other people. People can establish communication or express their purposes through speech, writing, music, sign language or even art. If establishing communication is the precondition for understanding, then expressing oneself is the basis of communication. In daily life verbal language is commonly used but there are instances when we can express ourselves mathematically or in symbols. In math we use certain expressions either verbally or by using symbols. Symbolic expressions can be a number or a combination of numbers using operational symbols. To further clarify our point we could say that 696 is a number while 3+12-4 is a combination of numbers where the addition and subtraction are operational symbols. Similarly, mathematical expressions could have unknown variables expressed by letters *a, b, c,…* or *x, y, z,…*and the like, as in the case 2x+3y-4z (Çağlar and Doğancıoğlu, 2000). In language classes, especially in grammar-based classes, parts of speech like nouns, verbs, adjectives, and prepositions are taught. These are, in fact, categories in a language just like elements are in chemistry, and are called "Categorial Grammar". When grammar is identified as such, it becomes possible to take the naming process a step further, and a more complicated set of categories based on the same ideas but that involve fractions, a term commonly used in mathematics comes to the forefront. Noun Phrases and Verb Phrases are needed to complete a sentence. If we were to express this in a fraction by coding Noun Phrases and Verb Phrases as NP and VP respectively and sentence with an S, we could come up with the following basic equation:

$$VP = S / NP$$

Then we could say that $VP \times NP = S$. Hence a verb phrase when put side by side by a Noun Phrase yields a sentence. The overall point we want to make here is that there is a mathematics of grammar; and hence, the analogy that chemistry and mathematics are categorized as grammar. However, the analogy is not absolutely true, for in chemistry or math, no matter who does the experiment in the chemistry lab or who solves the math problem, the results discovered for the former and found for the latter would be the same. In the case of categories of grammar, which is the foundation of language, discovery is more often art rather than a science.

The linguist is the one to set up the basics of grammar and people can discuss and argue about the choices. Hence comes the difference between the two, i.e., language and mathematics. But does this imply there is no interaction, no interdependency, no common points? In order to shed more light onto these points, the paper will elaborate on what mathematics is.

3. What is mathematics in the context of math education?

Humanity needs educated and creative people who can follow the accelerated technological developments and contribute to these developments. Mathematics is important for modern people to create and solve problems, think objectively and independently, be self confident, and explain the relations.

Mathematics is a language we use to identify, describe, and investigate the patterns and challenges of everyday living. It helps us to understand the events that have occurred, and to predict and prepare for events to come so that we can more fully understand our world, and more successfully live in it. To further support this assumption that basic understanding of mathematical concepts at problem solving level need to be made as easy as possible so that understanding of problems encountered in everyday life will not sound challenging and impossible to deal with if the educators, especially dealing with young learners and young adult learners, are cautious in the way they teach problem solving strategies so as not to cause the learners to fear math classes. Rather than resorting to difficult or complex ways in reaching the end, teachers should utilize simple solving techniques integrating "fun" activities into the math class. An example from the text book *Matematik Gezegeni , (*Çağlar, 1999:.202) illustrates our purpose very clearly:

> *3/8 of Ahmet's marbles is 36. When Ahmet gave ½ of his marbles to Ali, Ali ended up having 60 marbles. How many marbles did Ali have before Ahmet gave him the marbles?*

This problem can be solved using algebra in the following manner, however, such a method is not suitable for this age group ranging from 10-11:

a. Solution using algebra:

Let's assume Ahmet's marbles are represented by 'x'

Since 2/3 of Ahmet's marbles= 36

Then (3/8) . x= 36

x= 36. (8/3)

x= 96 marbles

½ of Ahmet's marbles =(½). X

= (1/2). 96

= 48

In order to find how many marbles Ali initially had, we have to subtract 48 from 60, which yields 12 marbles.

b. Proposed method of solving the problem:
36 / 3= 12 (each cell has 12 marbles)

12	12	12	12	12	12	12	12

Ahmet's marbles = 8 x 12 = 96
½ of Ahmet's marbles = 96 / 2 = 48
To find what Ali initially had: 60 – 48 = 12 marbles

In the *Matematik Gezegeni* math books, covering Kindergarten through Grade 8, such practical solutions that are articulate and easy to solve to all levels of learners are proposed. The underlying principle in these text books are Piaget's learning theories. (in Liebeck 1984)

It is important for teachers, educators and academicians to teach students, to make them enjoy and believe in the importance of mathematics, and help them to achieve a useful level of mathematics, which is the basis of the technological developments in our lives. The fact that mathematics gains importance from day to day shows that we have to give priority to teaching mathematical concepts in math education. Starting from childhood, concept development continuously becomes more important. At the beginning of this period, people establish relationships between the events and situations that tend to fall at a conflict with their lives and experiences.

Özçelik (1988: 1-3) has pointed out that concept formation is accepted as one's grouping of facts, cases and things, and showing the same or similar reactions to the facts, cases or events which are in the same group. Developing concepts at the beginning of the human life, and with a more inductive approach is becoming more acceptable in the community. Generally, it is called the concept attainment. It can be seen that these concepts can be earned before school and especially during the planned education period which is the school education. During the school period it is beneficiary to use inductive approach besides deductive approach in order to gain concepts. During this period, for example, wordy explanations, introductions are used in order to gain concepts, and inductive approach is used not only to support this approach but to eliminate the weaknesses of it. The concepts that one has at a certain time are, on one hand, the main products of his cognitive development until that time, and, on the other hand, essential motivating power on his cognitive development in the future. Conceptual development is a strong indicator and a main prerequisite of cognitive development. Because of these peculiarities, it is important to follow the concept development and to make accelerated concept development.

Mathematics education is similar to a building construction. Mathematics cannot be learned before basic mathematical concepts are understood. Knowledge can disappear just like a building can disappear in a matter of seconds due to an earthquake or explosives placed to demolish it. Daily studies are very important. One has to practice continuously. Learning and reinforcement have to be done in a certain period of time, and the learning process must be used effectively. This process is similar to a person who does not eat for a week and tries to eat twenty-one meals all at once. Normally a person can eat twenty-one meals in one week but he cannot eat them in one day. Similarly, a student, instead of learning the concepts of a course in one term, tries to learn them over night, and hence cannot succeed in learning all of them.

A widespread analogy is that the minds of learners is like a tower made of bricks (Peterson, et al., 1988-89:42-46) Before higher ones are put, the infrastructure must be built. The education in applied basic training depends on this idea, which asserts that it is impossible to gain necessary problem solving and application skills of higher level concepts and skills unless one gains lower level concepts and skills. On the other hand, recent theories and researches of cognitive psychology show us that knowledge is gained as concepts or structure in the minds of learners. Learning is to connect a relation between old knowledge and a newly learned one, and this connection is facilitated under good guidance. Researches on mind and learning claim that lower and higher level learning concepts are rarely effective. For instance, for calculation capability, rather than lower level skills, it is necessary to have the relationships which are learned as parts of problem solving method to solve higher level mathematical problems. In mathematics education, both the processes top to bottom and vice versa exist simultaneously.

According to the Cockcroft Report (1981); mathematics involves a hierarchy of abstractions, and we cannot understand any mathematical concept without also understanding the concepts on which it depends lower in the hierarchy. When children learn mathematics, they need to play with and explore real objects that interest them. To solve real problems we need to understand mathematical concepts. The term concept is used in variety of ways in the literature. Nevertheless, despite the divergent use of concept, there is some commonality among the various interpretations of concept. According to Wilson (1966), having a concept of something is equal in effect to understanding the use of correlated term. A concept involves a set of objects determined by properties common to elements of the set. This construct of a concept is shared by Bruner, et al.. (1956) who considers a concept to be a category determined by a collection of defined attributes. According to Ausubel, et al.,(1978) concepts are any objects, events, situations or properties that possess common critical attributes and are designated in any given culture by some accepted sign or symbol Scientific and mathematical concepts are significantly different from everyday concepts and are notoriously difficult to learn. It has been noted (Yıldırım, 1988:42) that being isolated from absolute beings and physical events make mathematics an effective language and inseparable from science. It is known that mathematics is a language to think and thinking is required in each moment of life. It is introduced that the mathematical thinking is not different from the daily and scientific thinking. Of course, language itself is abstract and we communicate mathematics through language. The mathematical experience of a child, like all his experience, must progress through the sequence of abstraction E.L.P.S. (where, E is *experience* with physical objects, L is spoken *language* that describes the experience, P is *pictures* that represent the experience and S is written *symbols* that generalize the experience), categorized in Cockcroft Report (1981).

Concept formation is an on-going process that begins at the early stages of childhood, and continues throughout a person's entire life. It has been noted that human beings establish mental relationships between and within other beings, phenomena and situations that affect their lives and exhibit congruent reactions to these mentally typified in the same group (Özçelik,1988:1-3). This grouping and typification and the resulting congruent reactions are assumed to define symptoms of concept formation. Not all concepts developed during childhood are intelligible to the incident or social environment but they develop and evolve to attain social intelligibility. Social evolution gains momentum and acceleration especially with the advent of the school years, i.e., the years of formal education. Children begin to learn concepts formed within a certain period of time are not only direct product of the mental-cognitive development but also the foundation of the development to come. In short, it can safely be said that concept formation is a primary indicator of mental-cognitive development as well as pre-condition.

Baykul and Aşkar (1987: 2-9) generalize the thoughts about what mathematics is as:

1. mathematics is the operations of counting, computing, measuring, and drawing
2. mathematics is a language that uses some symbols
3. mathematics is a logical system which improves logical thinking of human beings
4. mathematics is a means of understanding the world and improving the environment which is lived in.

What are the components of mathematics? To enumerate briefly, the answer would turn out to be arithmetic, measurement, algebra, trigonometry, statistics, and probability; but in a basic and simple sense, mathematics is involved with numbers, shapes, quantities, and numerical operations. The core issue in mathematics is to understand and solve problems. Utilizing the basic operations like addition, subtraction, multiplication and division help solve everyday problems. Aptitude in the usage of these for operations lays the foundation for success in confronting problematic situations arising throughout life.

Keith Devlin, states;

it is all mathematics, everywhere we look around us. Whether we see it or not…the mathematics is always there. When we are introduced to projected geometry, we find that even art, perhaps the field considered the farthest from mathematics, has mathematics in it (Life by Numbers, 1998*)*

Demonstrating the ability to use the four basic operations help us make sense of the world as well. Discussing issues related to budgets, price comparisons on merchandise, and methods of charging/paying interest rates would all be impossible had we not had a sense of numbers and numerical operations. At a very young age children demonstrate knowledge of identifying whole numbers and develop their learning standards to the extent of using comparisons of quantities, percentages, and higher level mathematics.

More attention must be given to building and assessing concepts of operations. The standard on number sense and numeration is the first of three important components of teaching and learning about number (NCTM, 1989). Concepts of operations is the second and builds on number sense. The third of three number standards, whole-number computation, must build on the other two areas (Trafton, and Zawojewski, 1990: 18-22). According to Trafton and Zawojewski (1990) the mathematics curriculum in grades K-4 and 5-8 should include the following concepts:

1. Concepts of Whole-Number Operations: In grades K-4, the mathematics curriculum should include concepts of addition, subtraction, multiplication, and division of whole numbers so that students can:

- develop meaning for the operations by modeling and discussing a rich variety of problem situations;
- relate the mathematical language and symbolism of operations to problem situations and informal language;
- recognize that a wide variety of problem structures can be represented by a single operation;
- develop operation sense

2. Number Systems and Number Theory: In grades 5-8 the mathematics curriculum should include the study of number systems and number theory so that students can:

- understand and appreciate the need for numbers beyond the whole numbers;
- develop and use order relations for whole numbers, fractions, decimals, integers, and rational numbers;
- extend their understanding of whole-number operations to fractions, decimals, integers, and rational numbers;
- understand how the basic arithmetic operations are related to one another;
- develop and apply number theory concepts (e.g., primes, factors, and multiples) in real-world and mathematical problem situations (*ibid*).

Hurwitz (1990: 701-703) has pointed out that "students have memorized a recipe for a particular mathematical situation. The recipe is in symbolic form, and the students do not necessarily understand the significance of the symbols… Without comprehension of the substance behind the symbolism, the memorization of symbols is meaningless". We, as teachers and educators, have to give chance and opportunities to the students to express their approaches, both orally and in writing. They have to engage in mathematics as a human activity and they have to learn to work and study cooperatively in small groups to solve problems as well as argue for their approach amid conflicting ideas and strategies (Moody, 1990: 730-736). According to Atkin, and Karplus (1962) and Karplus (1977) the most effective instructional approach to help students improve their reasoning abilities is "learning cycle." Each type of learning cycle (descriptive, empirical-inductive, and hypothetical-deductive) consists of the phase exploration, term introduction and concept application. Lawson (1998) states:

> *The essential feature of the learning cycle is that its use allows students to explore a segment of nature, raise questions, and reveal prior conceptions/misconceptions and debate and test them, thus becoming aware of inadequacies in prior ideas and of the thinking patterns involved in testing ideas*.

4.Mathematics and Language

Just as language functions in organizing mental activity, and is so essential to social formation and individual construction of mathematical ideas, all mathematical experience is conditioned by language. Consequently linguistic reduction is inevitable in any mathematical construction to locate and condition broader cogitations.

Many contemporary writers on language do not view language as providing and unproblematic labeling of the world (Brown: 1994) Bertrand Russell's statement "analytical philosophy's notion of language picturing reality" is no longer an adequate metaphor to show how language functions.(Russell. 1914) When the child is exposed to the language, the child is not taught the grammatical or the syntactical rules of the language. He or she learns it through experiencing the objects in the environment. He or she starts to learn words to describe desires, emotions and needs. Once the informal exposure is complete, i.e., the native language is learned implicitly, they would begin the explicit exposure to language.

Similarly, mathematics is language that is taught explicitly just like any language concentrating on grammar, syntax, and vocabulary. However, explicit learners confront problems in understanding the subtleties of the language, its idioms, its way of saying. Implicit learners, on the other hand, do not have sufficient knowledge of the underlined structure rules, and syntax. Rene Thom (1973) in his article on "Modern mathematics: does it exist?" argues whether mathematics should be taught first implicitly then explicitly. He considers that formal definitions, structures, and symbolic descriptions comprise explicit language whereas implicit language learning is "by direct use as an alien child would naturally learn it if immersed into this linguistic society". (Rene Thom, 1973)

Robert Moore in his article "Making the transition to formal proof" states that students encountered some difficulties in learning mathematics when we look at it as a language (1994: 249-266.). He found seven major sources of difficulty:

- Students did not know or were unable to state definitions;
- They had little intuitive understanding of the concepts;
- They had inadequate concept images;
- They were unable or unwilling to generate and use their own examples;
- They did not know how to use definitions;
- They were unable to understand and use mathematical language and notation; and
- They did not know how to begin proof.

These are specific examples encountered in learning the "language of mathematics" at almost any stage of mathematical development. It needs to be clarified that whether it be learning a language or mathematics, the skills required in learning both are not very different. They are both learned with varying degrees of success and interest and this does not mean that learning mathematics or any other language is the domain of the gifted and talented. Through practice we can all learn mathematics; if the suitable conditions are created and provided to the students, learning will be an easy process and will be definite. Underexposure to "mathematics and language" hinders mathematical development as well as language skills therefore it becomes possible to say that "if you do not use it, you lose it" belief becomes valid for both math and language. Hence daily practice in reading, writing, and mathematics from an early age will facilitate learning at further stages.

The use of mathematical concepts or terms in everyday language may at times lead to ambiguities. For example, the word and the number "zero" bring different associations depending on the context it is used. If we are to consider zero meaning "nothing", then when we say "you mean nothing to me" and "you owe me nothing" the implications are different. The former is used in everyday language and could be uttered by a parent to a child or between spouses in times of anger where "nothing has a very negative meaning. However, in the latter case, the term "nothing" has a positive implication as it suggests that having no debt is a positive phenomena. Another example to illustrate our purpose would be in situations where we say, "I have depleted everything"; which means you have got nothing left. To put this in mathematical terms "you have zero of everything". But in mathematics "zero" does not mean "nothing" as there are "positive and negative infinities". When we listen to the weather forecast especially in winter, the weatherman says, "today it is expected -20°C in Erzurum, 0°C in Ankara, and +20°C in Izmir". The zero degree Celsius does not mean that there is *no* temperature in Ankara. Ankara is 20 degrees warmer than Erzurum while 20 degrees colder than Izmir (Caglar 1999). In order to improve the language skills of children, teachers tend to give writing activities at times based on observation. The writing tasks do not demand long sentence formation from children as such sentences do not imply sophisticated writing. The same is valid for mathematics. The ability to perform very long digit calculations is not a good indicator of mathematical sophistication. In order to enable children to become aware of their environment and use the basic language skills, the math teacher could ask the primary school children to keep a track of the amount of basic essentials consumed within the household per week, and the results obtained by each child would be different. The children should be asked to explain the reasons for these different responses. The next stage could be to ask them to cut down their weekly expenses by 20 percent and ask them to explain the measures they took in order to reduce weekly expenditures. Such a task enables the child to use the language for different purposes, to formulate a family budget and to learn how to economize in times of need.

Children need not be taught certain formulas or rules in order to solve problems they might encounter in their everyday lives. To achieve this, we as teachers should provide our students with real situations rather than create artificial ones and expect them to think and solve problems that are far-fetched. A real experience the writer of this paper had at Middle East Technical University Development-Foundation Schools Primary Section with the 4th graders quite clearly explains and exemplifies how children do not resort to any kind of mathematical formula in order to come up with the right answer. After a visit paid by the South African children to their school, these fourth graders were asked to find how far Cape Town was to Ankara. The only information they had was, based on what they were told by their visitors. The children said they could measure the distance between Ankara and Cape Town using a ruler on the world map they had in the classroom. Surely this did not indicate how far Cape Town was. The only factual information they had was the distance between Istanbul and Ankara in kilometers. Then they measured again on the same map the distance between these two cities, and compared orally using their related language to explain how much further away Cape Town was to Ankara compared to Istanbul. Although the fourth graders were unaware of the concepts ratio and proportion, they found the distance between Cape Town and Ankara by devising their own ways. When they compared the result they found with the information given in the Encyclopedia, they saw to their great surprise that the result they found differed only by fifty kilometers. They were overjoyed at being so close to the right answer. As a further step, they also calculated the speed of the airplane although they had no idea about the speed problems. Therefore using such approaches in the math classroom, children will be forced to think and use their imagination and language; hence rote learning is eradicated. Another suggestion is to audiotape or to videotape a mathematics class to see how students describe their thinking processes, how students develop the context through dialogue using introduced vocabulary, and enable the teacher to see the concrete experiences they have had with mathematical tools for learning.

5. Conclusion

Students and teachers both need to be cautious in the way they approach mathematics and language. In the language class, the teacher should begin with first things first, i.e., drill and practice should be given the priority at the early stages of learning, and vocabulary and structure, along with the other skills like reading, writing and speaking should come later. What is significant at this stage is the exposure to language. All these apply to mathematics learning, and if the steps followed in language learning are applied to mathematics learning, needless to say this discipline will not pose a threat to children, and moreover they would appreciate it and have a better and more extensive comprehension of mathematics as they would do with a second or a foreign language provided the correct methodologies are applied.

As a final word, we could say that education aims to provide literacy and this could be achieved through language, and if there is a correlation between language and mathematics as this paper tries to put forward, then all efforts should be made to treat mathematics as a language.

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